

Study Guide

Inverse Functions and Relations

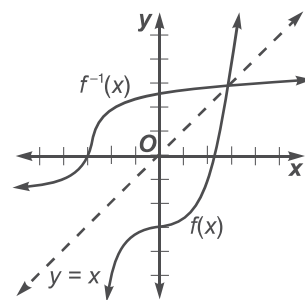
Two relations are inverse relations if and only if one relation contains the element (b, a) whenever the other relation contains the element (a, b) . If $f(x)$ denotes a function, then $f^{-1}(x)$ denotes the inverse of $f(x)$.

Example 1 Graph $f(x) = \frac{1}{4}x^3 - 3$ and its inverse.

To graph the function, let $y = f(x)$. To graph $f^{-1}(x)$, interchange the x - and y -coordinates of the ordered pairs of the function.

$f(x) = \frac{1}{4}x^3 - 3$	
x	y
-3	-9.75
-2	-5
-1	-3.25
0	-3
1	-2.75
2	-1
3	3.75

$f^{-1}(x)$	
x	y
-9.75	-3
-5	-2
-3.25	-1
-3	0
-2.75	1
-1	2
3.75	3



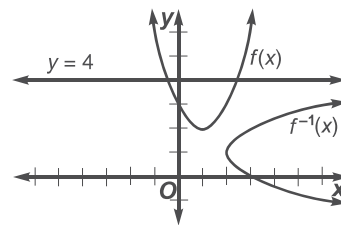
You can use the **horizontal line test** to determine if the inverse of a relation will be a function. If every horizontal line intersects the graph of the relation in at most one point, then the inverse of the relation is a function.

You can find the inverse of a relation algebraically. First, let $y = f(x)$. Then interchange x and y . Finally, solve the resulting equation for y .

Example 2 Find the inverse of $f(x) = (x - 1)^2 + 2$. Determine if the inverse is a function.

$$\begin{array}{ll}
 y = (x - 1)^2 + 2 & \text{Let } y = f(x). \\
 x = (y - 1)^2 + 2 & \text{Interchange } x \text{ and } y. \\
 x - 2 = (y - 1)^2 & \text{Isolate the expression containing } y. \\
 \pm\sqrt{x - 2} = y - 1 & \text{Take the square root of each side.} \\
 y = 1 \pm \sqrt{x - 2} & \text{Solve for } y. \\
 f^{-1}(x) = 1 \pm \sqrt{x - 2} & \text{Replace } y \text{ with } f^{-1}(x).
 \end{array}$$

Since the line $y = 4$ intersects the graph of $f(x)$ at more than one point, the function fails the horizontal line test. Thus, the inverse of $f(x)$ is not a function.

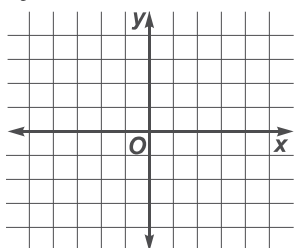


Practice

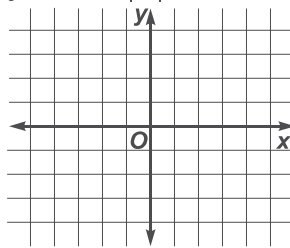
Inverse Functions and Relations

Graph each function and its inverse.

1. $f(x) = (x - 1)^3 + 1$



2. $f(x) = 3|x| + 2$

Find $f^{-1}(x)$. Then state whether $f^{-1}(x)$ is a function.

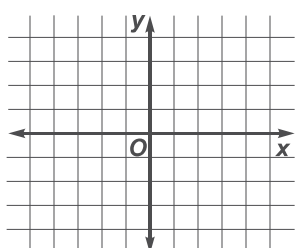
3. $f(x) = -4x^2 + 1$

4. $f(x) = \sqrt[3]{x - 1}$

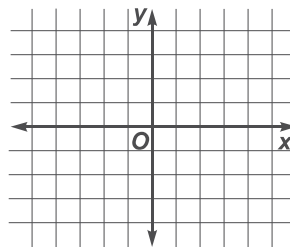
5. $f(x) = \frac{4}{(x - 3)^2}$

Graph each equation using the graph of the given parent function.

6. $y = -\sqrt{x + 3} - 1$, $p(x) = x^2$



7. $y = 2 + \sqrt[5]{x + 2}$, $p(x) = x^5$



8. **Fire Fighting** Airplanes are often used to drop water on forest fires in an effort to stop the spread of the fire. The time t it takes the water to travel from height h to the ground can be derived from the equation $h = \frac{1}{2}gt^2$ where g is the acceleration due to gravity (32 feet/second²).

a. Write an equation that will give time as a function of height.

b. Suppose a plane drops water from a height of 1024 feet. How many seconds will it take for the water to hit the ground?