



Summer Review Packet for Students Entering Pre-Calculus

Radicals:

To simplify means that 1) no radicand has a perfect square factor and
 2) there is no radical in the denominator (rationalize).

Recall – the **Product Property** $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$ and the **Quotient Property** $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$

Examples: Simplify $\sqrt{24} = \sqrt{4} \cdot \sqrt{6}$ find a perfect square factor
 $= 2\sqrt{6}$ simplify

Simplify $\sqrt{\frac{7}{2}} = \frac{\sqrt{7}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$ split apart, then multiply by both the numerator and the
 denominator by $\sqrt{2}$
 $= \frac{\sqrt{14}}{\sqrt{4}} = \frac{\sqrt{14}}{2}$ multiply straight across and simplify

If the denominator contains 2 terms –
 multiply the numerator and the denominator by the *conjugate* of the denominator
 The *conjugate* of $3 + \sqrt{2}$ is $3 - \sqrt{2}$ (the sign changes between the terms).

Simplify each of the following.

1. $\sqrt{32}$

2. $\sqrt{(2x)^8}$

3. $\sqrt[3]{-64}$

4. $\sqrt{49m^2n^8}$

5. $\sqrt{\frac{11}{9}}$

6. $\sqrt{60} \cdot \sqrt{105}$

7. $(\sqrt{5} - \sqrt{6})(\sqrt{5} + \sqrt{2})$

Rationalize.

8. $\frac{1}{\sqrt{2}}$

9. $\frac{2}{\sqrt{3}}$

10. $\frac{3}{2 - \sqrt{5}}$

Complex Numbers:Form of complex number: $a + bi$ Where a is the "real" part and bi is the "imaginary" partAlways make these substitutions: $i = \sqrt{-1}$, $i^2 = -1$, $i^3 = -\sqrt{-1}$ and $i^4 = 1$

- To simplify: pull out the $\sqrt{-1}$ before performing any operation

Example: $\sqrt{-5} = \sqrt{-1} \cdot \sqrt{5}$ Pull out $\sqrt{-1}$
 $= i\sqrt{5}$ Make substitution

Example: $(i\sqrt{5})^2 = i\sqrt{5} \cdot i\sqrt{5}$ List twice
 $= i^2 \sqrt{25}$ Simplify
 $= (-1)(5) = -5$ Substitute

- Treat i like any other variable when $+$, $-$, \times , or \div (but always simplify $i^2 = -1$)

Example: $2i(3 + i) = 2(3i) + 2i(i)$ Distribute
 $= 6i + 2i^2$ Simplify
 $= 6i + 2(-1)$ Make substitution
 $= -2 + 6i$ Simplify and rewrite in complex form

- Since $i = \sqrt{-1}$, no answer can have an ' i ' in the denominator, you must **RATIONALIZE!**

Simplify.

11. $\sqrt{-49}$

12. $6\sqrt{-12}$

13. $-6(2 - 8i) + 3(5 + 7i)$

14. $(3 - 4i)^2$

15. $(6 - 4i)(6 + 4i)$

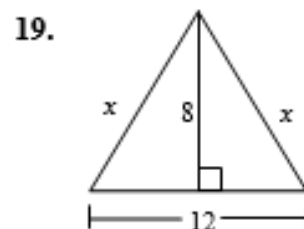
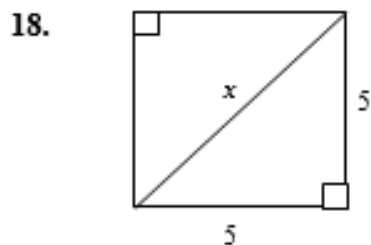
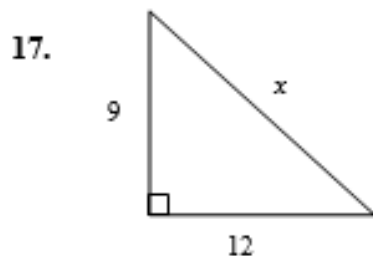
Rationalize.

16. $\frac{1+6i}{5i}$

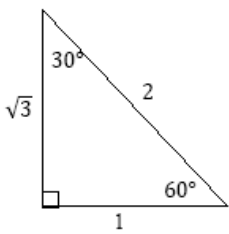
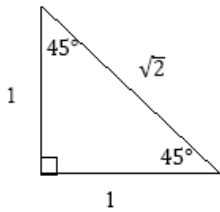
Geometry:

Pythagorean Theorem (right triangles): $a^2 + b^2 = c^2$

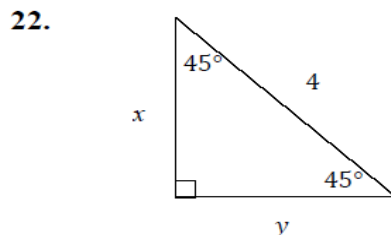
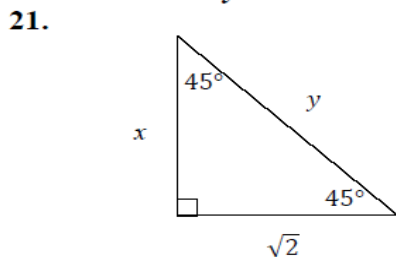
Find the value of x .



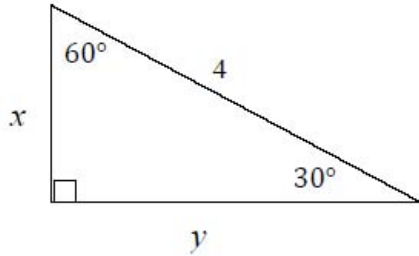
20. A square has perimeter 12 cm. Find the length of the diagonal.

In $30^\circ - 60^\circ - 90^\circ$ triangles, Sides are in proportion $1 : \sqrt{3} : 2$	In $45^\circ - 45^\circ - 90^\circ$ triangles, Sides are in proportion $1 : 1 : \sqrt{2}$
	

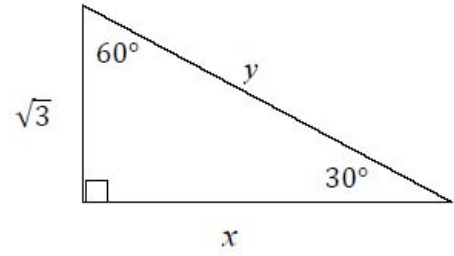
Solve for x and y .



23.



24.



Equations of Lines:

Slope intercept form: $y = mx + b$

Vertical line: $x = a$ (slope is undefined)

Point-slope form: $y - y_1 = m(x - x_1)$

Horizontal line: $y = b$ (slope is 0)

Standard Form: $Ax + By = C$

Slope: $m = \frac{y_2 - y_1}{x_2 - x_1}$

25. State the slope and y-intercept of the linear equation: $5x - 4y = 8$.

26. Find the x-intercept and y-intercept of the equation: $2x - y = 5$

27. Write the equation in standard form: $y = 7x - 5$

Write the equation of the line in slope-intercept form with the following conditions:

28. slope = -5 and passes through the point $(-3, -8)$

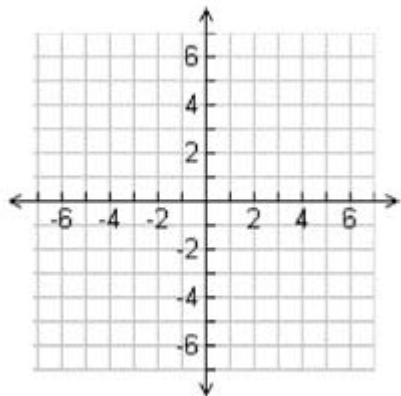
29. passes through the points $(4, 3)$ and $(7, -2)$

30. x-intercept = 3 and y-intercept = 2

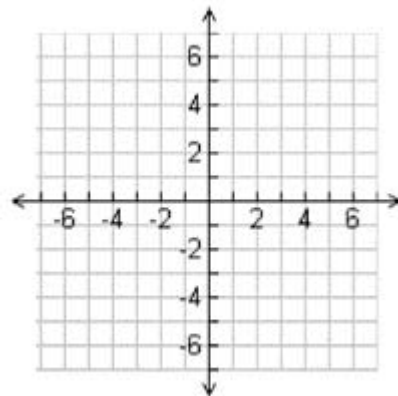
Graphing:

Graph each function, inequality, and / or system.

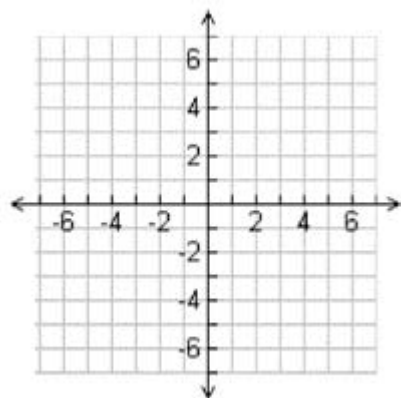
31. $3x - 4y = 12$



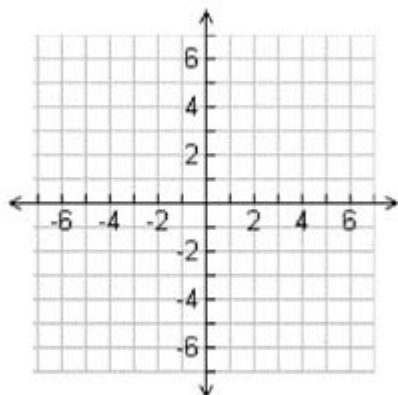
32.
$$\begin{cases} 2x + y = 4 \\ x - y = 2 \end{cases}$$



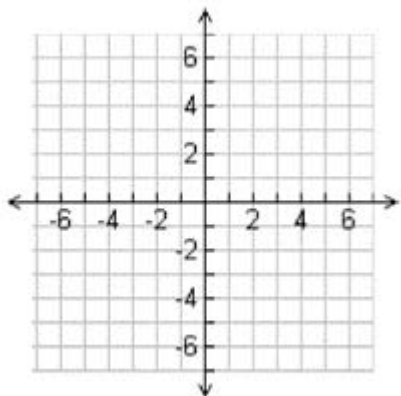
33. $y < -4x - 2$



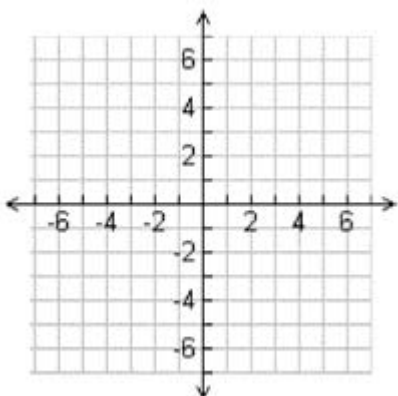
34. $y = \sqrt{x - 2}$



35. $y > |x| - 1$



36. $y + 4 = (x - 1)^2$



Vertex: _____
x-intercept(s): _____
y-intercept(s): _____

Systems of Equations:

$$3x + y = 6$$

$$2x - 2y = 4$$

Substitution:

Solve 1 equation for 1 variable.

Rearrange.

Plug into 2nd equation.

Solve for the other variable.

Then plug answer back into an original equation to solve for the 2nd variable.

$$y = 6 - 3x \quad \text{solve 1st equation for } y$$

$$2x - 2(6 - 3x) = 4 \quad \text{plug into 2nd equation}$$

$$2x - 12 + 6x = 4 \quad \text{distribute}$$

$$8x = 16 \quad \text{simplify}$$

$$x = 2$$

Plug $x = 2$ back into original

$$3(2) + y = 6$$

$$6 + y = 6$$

$$y = 0$$

Elimination:

Find opposite coefficients for 1 variable.

Multiply equation(s) by constant(s).

Add equations together (lose 1 variable).

Solve for variable.

$$6x + 2y = 12 \quad \text{multiply 1st equation by 2}$$

$$2x - 2y = 4 \quad \text{coefficients of } y \text{ are opposite}$$

$$8x = 16 \quad \text{add}$$

$$x = 2 \quad \text{simplify}$$

Solve each system of equations. Use any method.

$$37. \begin{cases} 2x + y = 4 \\ 3x + 2y = 1 \end{cases}$$

$$38. \begin{cases} 2x + y = 4 \\ 3x - y = 14 \end{cases}$$

$$39. \begin{cases} 2w - 5z = 13 \\ 6w + 3z = 10 \end{cases}$$

Exponents:

TWO RULES OF ONE

1. $a^1 = a$

Any number raised to the power of one equals itself.

2. $1^a = 1$

One to any power is one.

ZERO RULE

3. $a^0 = 1$

Any nonzero number raised to the power of zero is one.

PRODUCT RULE

4. $a^m \cdot a^n = a^{m+n}$

When multiplying two powers that have the same base, add the exponents.

QUOTIENT RULE

5. $\frac{a^m}{a^n} = a^{m-n}$

When dividing two powers with the same base, subtract the exponents.

POWER RULE

6. $(a^m)^n = a^{m \cdot n}$

When a power is raised to another power, multiply the exponents.

NEGATIVE EXPONENTS

7. $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Any nonzero number raised to a negative power equals its reciprocal raised to the opposite positive power.

Express each of the following in simplest form. Answers should not have any negative exponents.

40. $5a^0$

41. $\frac{3c}{c^{-1}}$

42. $\frac{2ef^{-1}}{e^{-1}}$

43. $\frac{(n^3 p^{-1})^2}{(np)^{-2}}$

Simplify.

44. $3m^2 \cdot 2m$

45. $(a^3)^2$

46. $(-b^3 c^4)^5$

47. $4m(3a^2 m)$

Polynomials:

To add / subtract polynomials, combine like terms.

$$\begin{aligned} \text{EX: } & 8x - 3y + 6 - (6y + 4x - 9) && \text{Distribute the negative through the parantheses.} \\ & = 8x - 3y + 6 - 6y - 4x + 9 && \text{Combine terms with similar variables.} \\ & = 8x - 4x - 3y - 6y + 6 + 9 \\ & = 4x - 9y + 15 \end{aligned}$$

Simplify.

48. $3x^3 + 9 + 7x^2 - x^3$

49. $7m - 6 - (2m + 5)$

To multiplying two binomials, use FOIL.

$$\begin{aligned} \text{EX: } & (3x - 2)(x + 4) && \text{Multiply the first, outer, inner, then last terms.} \\ & = 3x^2 + 12x - 2x - 8 && \text{Combine like terms.} \\ & = 3x^2 + 10x - 8 \end{aligned}$$

Multiply.

50. $(3a + 1)(a - 2)$

51. $(s + 3)(s - 3)$

52. $(c - 5)^2$

53. $(5x + 7y)(5x - 7y)$

Factoring

Follow these steps in order to factor polynomials.

STEP 1: Look for a GCF in ALL of the terms.

- If you have one (other than 1) factor it out front.
- If you don't have one, move on to STEP 2.

STEP 2: How many terms does the polynomial have?

2 Terms

a.) Is it difference of two squares? $a^2 - b^2 = (a + b)(a - b)$

EX: $x^2 - 25 = (x + 5)(x - 5)$

b.) Is it sum or difference of two cubes? $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
 $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

EX: $m^3 + 64 = (m + 4)(m^2 - 4m + 16)$

$$p^3 - 125 = (p - 5)(p^2 + 5p + 25)$$

3 Terms

$$x^2 + bx + c = (x + \quad)(x + \quad)$$

Ex: $x^2 + 7x + 12 = (x + 3)(x + 4)$

$$x^2 - bx + c = (x - \quad)(x - \quad)$$

$$x^2 - 5x + 4 = (x - 1)(x - 4)$$

$$x^2 + bx - c = (x - \quad)(x + \quad)$$

$$x^2 + 6x - 16 = (x - 2)(x + 8)$$

$$x^2 - bx - c = (x - \quad)(x + \quad)$$

$$x^2 - 2x - 24 = (x - 6)(x + 4)$$

4 Terms -- Factor by Grouping

- Pair up first two terms and last two terms
- Factor out GCF of each pair of numbers.
- Factor out front the parentheses that the terms have in common.
- Put leftover terms in parentheses.

$$\begin{aligned} \text{Ex: } x^3 + 3x^2 + 9x + 27 &= (x^3 + 3x^2) + (9x + 27) \\ &= x^2(x + 3) + 9(x + 3) \\ &= (x + 3)(x^2 + 9) \end{aligned}$$

Factor completely.

54. $z^2 + 4z - 12$

55. $6 - 5x - x^2$

56. $2k^2 + 2k - 60$

57. $-10b^4 - 15b^2$

58. $9c^2 + 30c + 25$

59. $9n^2 - 4$

60. $27z^3 - 8$

61. $2mn - 2mt + 2sn - 2st$

To solve quadratic equations, try to factor first and set each factor equal to zero. Solve for your variable. If the quadratic does NOT factor, use quadratic formula.

EX: $x^2 - 4x = 21$ *Set equal to zero FIRST.*

$x^2 - 4x - 21 = 0$ *Now factor.*

$(x + 3)(x - 7) = 0$ *Set each factor equal to zero.*

$x + 3 = 0$ $x - 7 = 0$ *Solve each for x.*

$x = -3$ $x = 7$

Solve each equation.

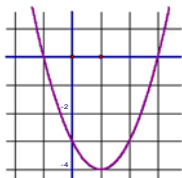
62. $x^2 - 4x - 12 = 0$

63. $x^2 + 25 = 10x$

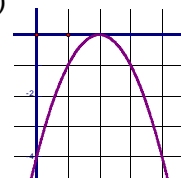
64. $x^2 - 14x + 40 = 0$

DISCRIMINANT: The number under the radical in the quadratic formula ($b^2 - 4ac$) can tell you what kinds of roots you will have.

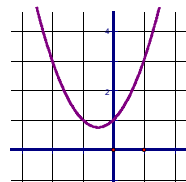
IF $b^2 - 4ac > 0$ **you will have TWO real roots.**
(touches x-axis twice)



IF $b^2 - 4ac = 0$ **you will have ONE real root**
(touches the x-axis once)



IF $b^2 - 4ac < 0$ **you will have TWO imaginary roots.**
(Graph does not cross the x-axis)



QUADRATIC FORMULA – allows you to solve any quadratic for all its real and imaginary

roots. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

EX: In the equation: $x^2 + 2x + 3 = 0$, find the value of the discriminant, describe the nature of the roots, then solve.

$x^2 + 2x + 3 = 0$ Determine values for a, b, and c.

$a = 1$ $b = 2$ $c = 3$ Find discriminant. $D = 2^2 - 4 \cdot 1 \cdot 3$

$D = 4 - 12$

$D = -8$ There are two imaginary roots.

Solve: $x = \frac{-2 \pm \sqrt{-8}}{2}$

$x = \frac{-2 \pm 2i}{2}$

$x = -1 \pm i$

Find the value of the discriminant, describe the nature of the roots, then solve each quadratic.

Use EXACT values.

65. $x^2 - 9x + 14 = 0$

66. $5x^2 - 2x + 4 = 0$

Discriminant = _____

Discriminant = _____

Type of Roots: _____

Type of Roots: _____

Roots = _____

Roots = _____

Long Division – can be used when dividing any polynomials.

Synthetic Division – can ONLY be used when dividing a polynomial by a linear (degree one) polynomial.

EX: $\frac{2x^3 + 3x^2 - 6x + 10}{x + 3}$

Long Division

$$\begin{array}{r} \frac{2x^3 + 3x^2 - 6x + 10}{x + 3} \\ 2x^2 - 3x + 3 + \frac{1}{x+3} \\ = x + 3 \overline{) 2x^3 + 3x^2 - 6x + 10} \\ \underline{(-) (2x^3 + 6x^2)} \\ -3x^2 - 6x \\ \underline{(-) (-3x^2 - 9x)} \\ 3x + 10 \\ \underline{(-) (3x + 9)} \\ 1 \end{array}$$

Synthetic Division

$$\begin{array}{r} \frac{2x^3 + 3x^2 - 6x + 10}{x + 3} \\ -3 \overline{) 2 \quad 3 \quad -6 \quad 10} \\ \quad \downarrow \quad -6 \quad 9 \quad -9 \\ \quad \hline \quad 2 \quad -3 \quad 3 \quad 1 \\ = 2x - 3x + 3 + \frac{1}{x+3} \end{array}$$

Divide each polynomial using long division OR synthetic division.

67. $\frac{c^3 - 3c^2 + 18c - 16}{c^2 + 3c - 2}$

68. $\frac{x^4 - 2x^2 - x + 2}{x + 2}$

To evaluate a function for a given value, simply plug the value into the function for x .

Evaluate each function for the given value.

69. $f(x) = x^2 - 6x + 2$

70. $g(x) = 6x - 7$

71. $f(x) = 3x^2 - 4$

$f(3) =$ _____

$g(x+h) =$ _____

$5[f(x+2)] =$ _____

Composition and Inverses of Functions:

Recall: $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read "f of g of x" Means to plug the inside function (in this case $g(x)$) in for x in the outside function (in this case, $f(x)$).

Example: Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$.

$$\begin{aligned} f(g(x)) &= f(x - 4) \\ &= 2(x - 4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Suppose $f(x) = 2x$, $g(x) = 3x - 2$, and $h(x) = x^2 - 4$. Find the following:

72. $f[g(2)] = \underline{\hspace{2cm}}$

73. $f[g(x)] = \underline{\hspace{2cm}}$

74. $f[h(3)] = \underline{\hspace{2cm}}$

75. $g[f(x)] = \underline{\hspace{2cm}}$

To find the inverse of a function, simply switch the x and the y and solve for the new "y" value.

Example:

$f(x) = \sqrt[3]{x+1}$	Rewrite f(x) as y	} Always the first three steps
$y = \sqrt[3]{x+1}$	Switch x and y	
$x = \sqrt[3]{y+1}$	Solve for your new y	
$(x)^3 = (\sqrt[3]{y+1})^3$	Cube both sides	
$x^3 = y + 1$	Simplify	
$y = x^3 - 1$	Solve for y	
$f^{-1}(x) = x^3 - 1$	Rewrite in inverse notation	

Find the inverse, $f^{-1}(x)$, if possible.

76. $f(x) = 5x + 2$

77. $f(x) = \frac{1}{2}x - \frac{1}{3}$

Rational Algebraic Expressions:

Multiplying and Dividing

Factor numerator and denominator completely. Cancel any common factors in the top and bottom. If dividing, change divide to multiply and flip the second fraction.

EX:

$$\frac{x^2 + 10x + 21}{5 - 4x - x^2} \cdot \frac{x^2 + 2x - 15}{x^3 + 4x^2 - 21x} \quad \text{Factor everything completely.}$$

$$= \frac{(x+7)(x+3)}{(5+x)(1-x)} \cdot \frac{(x+5)(x-3)}{x(x-3)(x+7)} \quad \text{Cancel out common factors in the top and bottom.}$$

$$= \frac{(x+3)}{x(1-x)} \quad \text{Simplify.}$$

Simplify.

$$78. \frac{5z^3 + z^2 - z}{3z}$$

$$79. \frac{m^2 - 25}{m^2 + 5m}$$

$$80. \frac{10r^5}{21s^2} \cdot \frac{3s}{5r^3}$$

$$81. \frac{a^2 - 5a + 6}{a+4} \cdot \frac{3a+12}{a-2}$$

$$82. \frac{6d-9}{5d+1} \div \frac{6-13d+6d^2}{15d^2-7d-2}$$

Addition and Subtraction.

First, find the least common denominator.

Write each fraction with the LCD.

Add / subtract numerators as indicated and leave the denominators as they are.

$$\text{EX: } \frac{3x+1}{x^2+2x} + \frac{5x-4}{2x+4}$$

Factor denominator completely.

$$= \frac{3x+1}{x(x+2)} + \frac{5x-4}{2(x+2)}$$

Find LCD (2x)(x+2)

$$= \frac{2(3x+1)}{2x(x+2)} + \frac{x(5x-4)}{2x(x+2)}$$

Rewrite each fraction with the LCD as the denominator.

$$= \frac{6x+2+5x^2-4x}{2x(x+2)}$$

Write as one fraction.

$$= \frac{5x^2+2x+2}{2x(x+2)}$$

Combine like terms.

Simplify.

83. $\frac{2x}{5} - \frac{x}{3}$

84. $\frac{b-a}{a^2b} + \frac{a+b}{ab^2}$

85. $\frac{2-a^2}{a^2+a} + \frac{3a+4}{3a+3}$

Complex Fractions.

Eliminate complex fractions by multiplying the numerator and denominator by the LCD of each of the small fractions. Then simplify as you did above

EX:

$$\frac{1 + \frac{1}{a}}{\frac{2}{a^2} - 1}$$

Find LCD : a^2

$$= \frac{\left(1 + \frac{1}{a}\right) ga^2}{\left(\frac{2}{a^2} - 1\right) ga^2}$$

Multiply top and bottom by LCD.

$$= \frac{a^2 + a}{2 - a^2}$$

Factor and simplify if possible.

$$= \frac{a(a+1)}{2 - a^2}$$

$$86. \frac{1 - \frac{1}{2}}{2 + \frac{1}{4}}$$

$$87. \frac{1 + \frac{1}{z}}{z + 1}$$

$$88. \frac{5 + \frac{1}{m} - \frac{6}{m^2}}{\frac{2}{m} - \frac{2}{m^2}}$$

$$89. \frac{2 + \frac{1}{x} - \frac{1}{x^2}}{1 + \frac{4}{x} + \frac{3}{x^2}}$$

Solving Rational Equations:

Multiply each term by the LCD of all the fractions. This should eliminate all of your fractions. Then solve the equation as usual.

$$\frac{5}{x+2} + \frac{1}{x} = \frac{5}{x} \quad \text{Find LCD first. } x(x+2)$$

$$x(x+2)\left(\frac{5}{x+2}\right) + x(x+2)\left(\frac{1}{x}\right) = \left(\frac{5}{x}\right)x(x+2) \quad \text{Multiply each term by the LCD.}$$

$$5x + 1(x+2) = 5(x+2) \quad \text{Simplify and solve.}$$

$$5x + x + 2 = 5x + 10$$

$$6x + 2 = 5x + 10$$

EX: $x = 8$ \leftarrow Check your answer. Sometimes they do not check!

Check:

$$\frac{5}{8+2} + \frac{1}{8} = \frac{5}{8}$$

$$\frac{5}{10} + \frac{1}{8} = \frac{5}{8}$$

$$\frac{5}{8} = \frac{5}{8}$$

Solve each equation. Check your solutions.

90. $\frac{12}{x} + \frac{3}{4} = \frac{3}{2}$

91. $\frac{x+10}{x^2-2} = \frac{4}{x}$

92. $\frac{5}{x-5} = \frac{x}{x-5} - 1$

LOGARITHMS

PROPERTIES OF LOGARITHMS $\log_a x$ ($a > 0, a \neq 1$)		
<i>Logarithm with Base a</i>		<i>Natural Logarithm</i>
$\log_a 1 = 0$		$\ln 1 = 0$
$\log_a a = 1$		$\ln e = 1$
$\log_a mn = \log_a m + \log_a n$		$\ln mn = \ln m + \ln n$
$\log_a \frac{m}{n} = \log_a m - \log_a n$		$\ln \frac{m}{n} = \ln m - \ln n$
$\log_a x^m = m \log_a x$		$\ln x^m = m \ln x$
$a^x = a^y$ if and only if $x = y$		$e^x = e^y$ if and only if $x = y$
If $\log_a x = \log_a y$ then $x = y$		If $\ln x = \ln y$ then $x = y$
$\log_a a^x = x$		$\ln e^x = x$
$a^{\log_a x} = x$		$e^{\ln x} = x$
Change of Base Formula	$\log_a x = \frac{\log_b x}{\log_b a}$	$\log_a x = \frac{\ln x}{\ln a}$ Change of Base Formula

Use properties of logarithms to expand the expressions. Simplify, if possible.

93. $\log_3 9x$

94. $\ln \sqrt[3]{64m}$

95. $\log_2 \frac{3x^2}{12xy}$

Use properties of logarithms to condense the expressions. Simplify, if possible.

96. $3 \log_2 x + 5 \log_2 y - 2 \log_2 z$

97. $\frac{1}{2}(\ln x + \ln 4 - \ln 9)$

Solve, round answer to three decimal places.

98. $3^x = 20$

99. $8 + 2 \ln x = 5$

100. $e^{2x} = 12$

101. $500 e^{-x} = 200$

102. $-13 + 3e^x = 9$