Solving Exponential & Logarithmic Equations

NOTES ¿ Assistance

- **Properties of Exponential and Logarithmic Equations**

  Let a be a positive real number such that $a \neq 1$, and let x and y be real numbers. Then the following properties are true:

  1. $a^x = a^y$ if and only if $x = y$
  2. $\log_a x = \log_a y$ if and only if $x = y$ $(x > 0,\ y > 0)$

- **Inverse Properties of Exponents and Logarithms**

	Base a	*Natural Base e*
1.	$\log_a(a^x) = x$	$\ln(e^x) = x$
2.	$a^{(\log_a x)} = x$	$e^{(\ln x)} = x$

- **Solving Exponential and Logarithmic Equations**

  1. To solve an exponential equation, first isolate the exponential expression, then **take the logarithm of both sides of the equation** and solve for the variable.
  2. To solve a logarithmic equation, first isolate the logarithmic expression, then **exponentiate both sides of the equation** and solve for the variable.

     For Instance: If you wish to solve the equation, $\ln x = 2$, you **exponentiate** both sides of the equation to solve it as follows:

$\ln x = 2$	Original equation
$e^{\ln x} = e^2$	Exponentiate both sides
$x = e^2$	Inverse property

Or you can simply rewrite the logarithmic equation in exponential form to solve (i.e. $\ln x = 2$ if and only if $e^2 = x$).

Note: You should *always* check your solution in the original equation.

Example 1:

Solve each equation.

a. $4^{x+2} = 64$

b. $\ln(2x-3) = \ln 11$

Solution:

a.

$4^{x+2} = 64$	Original Equation
$4^{x+2} = 4^3$	Rewrite with like bases
$x + 2 = 3$	Property of exponential equations
$x = 1$	Subtract 2 from both sides

The solution is 1. Check this in the original equation.

b.

$\ln(2x-3) = \ln 11$	Original Equation
$2x - 3 = 11$	Property of logarithmic equations
$2x = 14$	Add 3 to both sides
$x = 7$	Divide both sides by 2

The solution is 7. Check this in the original equation.

Example 2:

Solve $5 + e^{x+1} = 20$.

Solution:

$5 + e^{x+1} = 20$	Original Equation
$e^{x+1} = 15$	Subtract 5 from both sides
$\ln e^{x+1} = \ln 15$	Take the logarithm of both sides
$x + 1 = \ln 15$	Inverse Property
$x = -1 + \ln 15 \approx 1.708$	Subtract 1 from both sides

Check:

$5 + e^{x+1} = 20$	Original Equation
$5 + e^{1.708+1} \overset{?}{=} 20$	Substitute 1.708 for x
$5 + e^{2.708} \overset{?}{=} 20$	Simplify
$5 + 14.999 \approx 20$	Solution checks ✓

Example 3:

Solve the exponential equations.

a. $2^x = 7$ b. $4^{x-3} = 9$ c. $2e^x = 10$

Solutions:

Method 1:

a. $2^x = 7$ Original Equation

$\log 2^x = \log 7$ Take the logarithm of both sides

$x(\log 2) = \log 7$ Property of Logarithms

$x = \frac{\log 7}{\log 2} \approx 2.807$ Solve for x

Method 2:

a. $2^x = 7$ Original Equation

$\log_2 2^x = \log_2 7$ Take the logarithm of both sides

$x = \log_2 7$ Inverse Property

$x = \frac{\log 7}{\log 2} \approx 2.807$ Change of Base Formula

Method 1:

b. $4^{x-3} = 9$ Original Equation

$\log 4^{x-3} = \log 9$ Take the logarithm of both sides

$(x-3)\log 4 = \log 9$ Property of Logarithms

$x - 3 = \frac{\log 9}{\log 4}$ Divide both sides by log 4

$x = 3 + \frac{\log 9}{\log 4} \approx 4.585$ Solve for x

Method 2:

b. $4^{x-3} = 9$ Original Equation

$\log_4 4^{x-3} = \log_4 9$ Take the logarithm of both sides

$x - 3 = \log_4 9$ Inverse Property

$x - 3 = \frac{\log 9}{\log 4}$ Change of Base Formula

$x = 3 + \frac{\log 9}{\log 4} \approx 4.585$ Solve for x

c. $2e^x = 10$ Original Equation

$e^x = 5$ Divide both sides by 2

$\ln e^x = \ln 5$ Take the logarithm of both sides

$x = \ln 5 \approx 1.609$ Inverse Property

Example 4:

Solve $2\log_4 x = 5$.

Solution:

$2\log_4 x = 5$ Original Equation

$\log_4 x = \frac{5}{2}$ Divide both sides by 2

$4^{5/2} = x$ Change to exponential form

$x = 32$ Simplify

Example 5:

Solve $3\log x = 6$.

Solution:

$3\log x = 6$ Original Equation

$\log x = 2$ Divide both sides by 3

$10^2 = x$ Change to exponential form

$x = 100$ Simplify

Example 6:

Solve $20\ln 0.2x = 30$.

Solution:

$20\ln 0.2x = 30$ Original Equation

$\ln 0.2x = 1.5$ Divide both sides by 20

$0.2x = e^{1.5}$ Change to exponential form

$x = 5e^{1.5} \approx 22.408$ Divide both sides by 0.2

Example 7: Solving a Logarithmic Equation using Exponentiation

Solve $\log_3 2x - \log_3(x-3) = 1$

Solution:

$\log_3 2x - \log_3(x-3) = 1$ Original Equation

$\log_3 \frac{2x}{x-3} = 1$ Condense the left side

$3^{\log_3 \frac{2x}{x-3}} = 3^1$ Exponentiate both sides

$\frac{2x}{x-3} = 3$ Inverse Property

$2x = 3x - 9$ Multiply both sides by $x - 3$

$x = 9$ Solve for x

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Study Guide

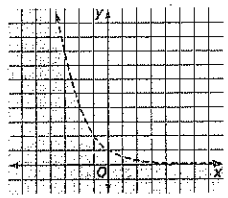
Exponential Functions

Functions of the form $y = b^x$, in which the base b is a positive real number and the exponent is a variable, are known as **exponential functions.** Many real-world situations can be modeled by exponential functions. The equation $N = N_0(1 + r)^t$, where N is the final amount, N_0 is the initial amount, r is the rate of growth or decay, and t is time, is used for modeling exponential growth. The compound interest equation is $A = P\left(1 + \frac{r}{n}\right)^{nt}$, where P is the principal or initial investment, A is the final amount of the investment, r is the annual interest rate, n is the number of times interest is compounded each year, and t is the number of years.

Example 1 **Graph $y < 2^{-x}$.**

First, graph $y = 2^{-x}$. This graph is a function, since there is a unique y-value for each x-value.

x	−3	−2	−1	0	1	2	3	4
2^{-x}	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$



Since the points on this curve are not in the solution of the inequality, the graph of $y = 2^{-x}$ is shown as a dashed curve.

Then, use (0, 0) as a test point to determine which area to shade.

$y < 2^{-x}$
$0 < 2^0$
$0 < 1$

Since (0,0) satisfies the inequality, the region that contains (0,0) should be shaded.

Example 2 ***Biology*** **Suppose a researcher estimates that the initial population of a colony of cells is 100. If the cells reproduce at a rate of 25% per week, what is the expected population of the colony in six weeks?**

$N = N_0(1 + r)^t$
$N = 100(1 + 0.25)^6$ *$N_0 = 100$, $r = 0.25$, $t = 6$*
$N \approx 381.4697266$ *Use a calculator.*

There will be about 381 cells in the colony in 6 weeks.

Example 3 ***Finance*** **Determine the amount of money in a money market account that provides an annual rate of 6.3% compounded quarterly if $1700 is invested and left in the account for eight years.**

$A = P\left(1 + \frac{r}{n}\right)^{nt}$
$A = 1700\left(1 + \frac{0.063}{4}\right)^{4 \cdot 8}$ *$P = 1700$, $r = 0.063$, $n = 4$, $t = 8$*
$A \approx 2803.028499$ *Use a calculator.*

After 8 years, the $1700 investment will have a value of $2803.03.

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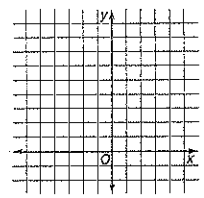
Practice

Exponential Functions

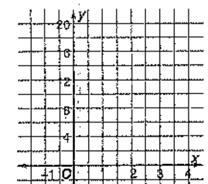
Extra Credit!

Graph each exponential function or inequality.

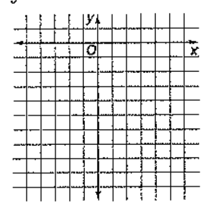
1. $y = 2^{x-1}$



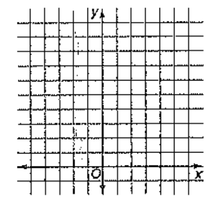
2. $y = 4^{-x+2}$



3. $y > -3^x + 1$



4. $y \geq 0.5^x$



5. ***Demographics*** An area in North Carolina known as The Triangle is principally composed of the cities of Durham, Raleigh, and Chapel Hill. The Triangle had a population of 700,000 in 1990. The average yearly rate of growth is 5.9%. Find the projected population for 2010.

6. ***Finance*** Determine the amount of money in a savings account that provides an annual rate of 4% compounded monthly if the initial investment is $1000 and the money is left in the account for 5 years.

7. ***Investments*** How much money must be invested by Mr. Kaufman if he wants to have $20,000 in his account after 15 years? He can earn 5% compounded quarterly.

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Study Guide

The Number e

The number e is a special irrational number with an approximate value of 2.718 to three decimal places. The formula for exponential growth or decay is $N = N_0e^{kt}$, where N is the final amount, N_0 is the initial amount, k is a constant, and t is time. The equation $A = Pe^{rt}$, where P is the initial amount, A is the final amount, r is the annual interest rate, and t is time in years, is used for calculating interest that is compounded continuously.

Example 1 ***Demographics*** **The population of Dubuque, Iowa, declined at a rate of 0.4% between 1997 1998. In 1998, the population was 87,806.**

a. Let t be the number of years since 1998 and write a function to model the population.

b. Suppose that the rate of decline remains steady at 0.4%. Find the projected population of Dubuque in 2010.

a. $y = ne^{kt}$
$y = 87{,}806e^{-0.004t}$ *$n = 87{,}806$; $k = -0.004$*

b. In 2010, it will have been 2010 − 1998 or 12 years since the initial population figure. Thus, $t = 12$.

$y = 87{,}806e^{\ 0.004t}$
$y = 87{,}806e^{-0.004(12)}$ *$t = 12$*
$y \approx 83690.86531$ *Use a calculator.*

Given a population of 87,806 in 1998 and a steady rate of decline of 0.4%, the population of Dubuque, Iowa, will be approximately 83,691 in 2010.

Example 2 ***Finance*** **Compare the balance after 10 years of a $5000 investment earning 8.5% interest compounded continuously to the same investment compounded quarterly.**

In both cases, $P = 5000$, $r = 0.085$, and $t = 10$. When the interest is compounded quarterly, $n = 4$. Use a calculator to evaluate each expression.

Continuously	Quarterly
$A = Pe^{rt}$	$A = P\left(1 + \frac{r}{n}\right)^{nt}$
$A = 5000e^{(0.085)(10)}$	$A = 5000\left(1 + \frac{0.085}{4}\right)^{4 \cdot 10}$
$A = 11{,}698.23$	$A = 11{,}594.52$

You would earn $11,698.23 − $11,594.52 = $103.71 more by choosing the account that compounds continuously.

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Practice

The Number e

1. ***Demographics*** In 1995, the population of Kalamazoo, Michigan, was 79,089. This figure represented a 0.4% annual decline from 1990.
   a. Let t be the number of years since 1995 and write a function that models the population in Kalamazoo in 1995.

   b. Predict the population in 2010 and 2015. Assume a steady rate of decline.

2. ***Biology*** Suppose a certain type of bacteria reproduces according to the model $P(t) = 100e^{0.271t}$, where t is time in hours.
   a. At what rate does this type of bacteria reproduce?

   b. What was the initial number of bacteria?

   c. Find the number of bacteria at $P(5)$, $P(10)$, $P(24)$, and $P(72)$. Round to the nearest whole number.

3. ***Finance*** Suppose Karyn deposits $1500 in a savings account that earns 6.75% interest compounded continuously. She plans to withdraw the money in 6 years to make a $2500 down payment on a car. Will there be enough funds in Karyn's account in 6 years to meet her goal?

4. ***Banking*** Given the original principal, the annual interest rate, the amount of time for each investment, and the type of compounded interest, find the amount at the end of the investment.
   a. $P = \$1250$, $r = 8.5\%$, $t = 3$ years, semiannually

   b. $P = \$2575$, $r = 6.25\%$, $t = 5$ years 3 months, continuously

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Study Guide

Logarithmic Functions

In the function $x = a^y$, y is called the **logarithm** of x. It is usually written as $y = \log_a x$ and is read "y equals the log, base a, of x." Knowing that if $a^u = a^v$ then $u = v$, you can evaluate a logarithmic expression to determine its logarithm.

Example 1 **Write $\log_7 49 = 2$ in exponential form.**

The base is 7 and the exponent is 2.
$7^2 = 49$

Example 2 **Write $2^5 = 32$ in logarithmic form.**

The base is 2, and the exponent or logarithm is 5.
$\log_2 32 = 5$

Example 3 **Evaluate the expression $\log_5 \frac{1}{25}$.**

Let $x = \log_5 \frac{1}{25}$.

$x = \log_5 \frac{1}{25}$
$5^x = \frac{1}{25}$ — *Definition of logarithm.*
$5^x = (25)^{-1}$ — $a^{-m} = \frac{1}{a^m}$
$5^x = (5^2)^{-1}$ — $5^2 = 25$
$5^x = 5^{-2}$ — $(a^m)^n = a^{mn}$
$x = -2$ — *If $a^u = a^v$, then $u = v$.*

Example 4 **Solve each equation.**

a. $\log_6 (4x + 6) = \log_6 (8x - 2)$

$\log_6 (4x + 6) = \log_6 (8x - 2)$
$4x + 6 = 8x - 2$ — *If $\log_b m = \log_b n$, then $m = n$.*
$-4x = -8$
$x = 2$

b. $\log_9 x + \log_9 (x - 2) = \log_9 3$

$\log_9 x + \log_9 (x - 2) = \log_9 3$
$\log_9 [x(x - 2)] = \log_9 3$ — *$\log_b mn = \log_b m + \log_b n$*
$x^2 - 2x = 3$ — *If $\log_b m = \log_b n$, then $m = n$.*
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$ — *Factor.*
$x - 3 = 0$ or $x + 1 = 0$ — *Find the zeros.*
$x = 3$ or $x = -1$.

The log of a negative value does not exist, so the answer is $x = 3$.

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Practice

Logarithmic Functions

Write each equation in exponential form.

1. $\log_3 81 = 4$
2. $\log_8 2 = \frac{1}{3}$
3. $\log_{10} \frac{1}{100} = -2$

Write each equation in logarithmic form.

4. $3^3 = 27$
5. $5^{-3} = \frac{1}{125}$
6. $\left(\frac{1}{4}\right)^{-4} = 256$

Evaluate each expression.

7. $\log_7 7^3$
8. $\log_{10} 0.001$
9. $\log_8 4096$
10. $\log_4 32$
11. $\log_3 1$
12. $\log_6 \frac{1}{216}$

Solve each equation.

13. $\log_x 64 = 3$
14. $\log_4 0.25 = x$
15. $\log_4 (2x - 1) = \log_4 16$
16. $\log_{10} \sqrt{10} = x$
17. $\log_7 56 - \log_7 x = \log_7 4$
18. $\log_5 (x + 4) + \log_5 x = \log_5 12$
19. ***Chemistry*** How long would it take 100,000 grams of radioactive iodine, which has a half-life of 60 days, to decay to 25,000 grams? Use the formula $N = N_0\left(\frac{1}{2}\right)^t$, where N is the final amount of the substance, N_0 is the initial amount, and t represents the number of half-lives.

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Study Guide

Common Logarithms

Logarithms with base 10 are called **common logarithms.** The change of base formula, $\log_a n = \frac{\log_b n}{\log_b a}$, where a, b, and n are positive numbers and neither a nor b is 1, allows you to evaluate logarithms in other bases with a calculator. Logarithms can be used to solve **exponential equations.**

Example 1 **Evaluate each expression.**

a. $\log 8(3)^2$

$$\begin{aligned} \log 8(3)^2 &= \log 8 + 2\log 3 && \textit{log ab = log a + log b, log } b^n = n \textit{ log b} \\ &\approx 0.9031 + 2(0.4771) && \textit{Use a calculator.} \\ &\approx 0.9031 + 0.9542 \\ &\approx 1.8573 \end{aligned}$$

b. $\log \frac{15^3}{7}$

$$\begin{aligned} \log \frac{15^3}{7} &= 3\log 15 - \log 7 && \log \tfrac{a}{b} = \log a - \log b,\ \log a^m = m \log a \\ &\approx 3(1.1761) - 0.8451 && \textit{Use a calculator.} \\ &\approx 3.5283 - 0.8451 \\ &\approx 2.6832 \end{aligned}$$

Example 2 **Find the value of $\log_8 2037$ using the change of base formula.**

$$\begin{aligned} \log_8 2037 &= \frac{\log_{10} 2037}{\log_{10} 8} && \log_a n = \frac{\log_b n}{\log_b a} \\ &\approx \frac{3.3090}{0.9031} && \textit{Use a calculator.} \\ &\approx 3.6641 \end{aligned}$$

Example 3 **Solve $7^{2x} = 93$.**

$$\begin{aligned} 7^{2x} &= 93 \\ \log 7^{2x} &= \log 93 && \textit{Take the logarithm of each side.} \\ 2x \log 7 &= \log 93 && \log_b m^p = p \cdot \log_b m \\ 2x &= \frac{\log 93}{\log 7} && \textit{Divide each side by log 7.} \\ 2x &\approx 2.3293 && \textit{Use a calculator.} \\ x &\approx 1.1646 \end{aligned}$$

11-5

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Practice

Common Logarithms

Given that log 3 = 0.4771, log 5 = 0.6990, and log 9 = 0.9542, evaluate each logarithm.

1. $\log 300{,}000$
2. $\log 0.0005$
3. $\log 9000$
4. $\log 27$
5. $\log 75$
6. $\log 81$

Evaluate each expression.

7. $\log 66.3$
8. $\log \frac{17^4}{5}$
9. $\log 7(4^3)$

Find the value of each logarithm using the change of base formula.

10. $\log_6 832$
11. $\log_{11} 47$
12. $\log_3 9$

Solve each equation or inequality.

13. $8^x = 10$
14. $2.4^x \leq 20$
15. $1.8^{x-5} = 19.8$
16. $3^{5x} = 85$
17. $4^{2x} > 25$
18. $3^{2x-2} = 2^x$

19. ***Seismology*** The intensity of a shock wave from an earthquake is given by the formula $R = \log_{10} \frac{I}{I_0}$, where R is the magnitude, I is a measure of wave energy, and $I_0 = 1$. Find the intensity per unit of area for the following earthquakes.

a. Northridge, California, in 1994, $R = 6.7$

b. Hector Mine, California, in 1999, $R = 7.1$

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11-6

Study Guide

Natural Logarithms

Logarithms with base e are called **natural logarithms** and are usually written **ln *x***. Logarithms with a base other than e can be converted to natural logarithms using the change of base formula. The antilogarithm of a natural logarithm is written **antiln *x***. You can use the properties of logarithms and antilogarithms to simplify and solve exponential and logarithmic equations or inequalities with natural logarithms.

Example 1 **Convert $\log_4 381$ to a natural logarithm and evaluate.**

$$\log_a n = \frac{\log_b n}{\log_b a}$$

$$\log_4 381 = \frac{\log_e 381}{\log_e 4} \qquad a = 4,\ b = e,\ n = 381$$

$$= \frac{\ln 381}{\ln 4} \qquad \log_e x = \ln x$$

$$\approx 4.2868 \qquad \textit{Use a calculator.}$$

So, $\log_4 381$ is about 4.2868.

Example 2 **Solve $3.75 = -7.5 \ln x$.**

$$3.75 = -7.5 \ln x$$

$$-0.5 = \ln x \qquad \textit{Divide each side by } -7.5$$

$$\text{antiln}(-0.5) = x \qquad \textit{Take the antilogarithm of each side.}$$

$$0.6065 \approx x \qquad \textit{Use a calculator.}$$

The solution is about 0.6065.

Example 3 **Solve each equation or inequality by using natural logarithms.**

a. $4^{3x} = 6^{x+1}$

$$4^{3x} = 6^{x+1}$$

$$\ln 4^{3x} = \ln 6^{x+1} \qquad \textit{Take the natural logarithm of each side.}$$

$$3x \ln 4 = (x + 1) \ln 6 \qquad \ln a^n = n \ln a$$

$$3x(1.3863) = (x + 1)(1.7918) \qquad \textit{Use a calculator.}$$

$$4.1589x = 1.7918x + 1.7918$$

$$2.3671x = 1.7918$$

$$x \approx 0.7570$$

b. $25 > e^{0.2t}$

$$25 > e^{0.2t}$$

$$\ln 25 > \ln e^{0.2t} \qquad \textit{Take the natural logarithm of each side.}$$

$$\ln 25 > 0.2t \ln e \qquad \ln a^n = n \ln a$$

$$3.2189 > 0.2t \qquad \textit{Use a calculator.}$$

$$16.0945 > t$$

Thus, $t < 16.0945$

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11-6

Practice

Natural Logarithms

Evaluate each expression.

1. $\ln 71$
2. $\ln 8.76$
3. $\ln 0.532$
4. $\text{antiln } -0.256$
5. $\text{antiln } 4.62$
6. $\text{antiln } -1.62$

Convert each logarithm to a natural logarithm and evaluate.

7. $\log_7 94$
8. $\log_5 256$
9. $\log_9 0.712$

Use natural logarithms to solve each equation or inequality.

10. $6^x = 42$
11. $7^x = 4^{x+3}$
12. $1249 = 175e^{0.04t}$
13. $10^{x+1} > 3^x$
14. $12 < e^{0.048y}$
15. $8.4 < e^{t-2}$

16. *Banking* Ms. Cubbatz invested a sum of money in a certificate of deposit that earns 8% interest compounded continuously. The formula for calculating interest that is compounded continuously is $A = Pe^{rt}$. If Ms. Cubbatz made the investment on January 1, 1995, and the account was worth $12,000 on January 1, 1999, what was the original amount in the account?

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Study Guide

Modeling Real-World Data with Exponential and Logarithmic Functions

The **doubling time,** or amount of time t required for a quantity modeled by the exponential equation $N = N_0e^{kt}$ to double, is given by $t = \frac{\ln 2}{k}$.

Example ***Finance*** **Tara's parents invested $5000 in an account that earns 11.5% compounded continuously. They would like to double their investment in 5 years to help finance Tara's college education.**

a. Will the initial investment of $5000 double within 5 years?

Find the doubling time for the investment. For continuously compounded interest, the constant k is the interest rate written as a decimal.

$t = \frac{\ln 2}{k}$

$= \frac{\ln 2}{0.115}$ *The decimal for 11.5% is 0.115.*

≈ 6.03 years *Use a calculator.*

Five years is not enough time for the initial investment to double.

b. What interest rate is required for an investment with continuously compounded interest to double in 5 years?

$t = \frac{\ln 2}{k}$

$5 = \frac{\ln 2}{k}$

$\frac{1}{5} = \frac{k}{\ln 2}$ *Take the reciprocal of each side.*

$\frac{\ln 2}{5} = k$ *Multiply each side by ln 2 to solve for k.*

$0.1386 \approx k$

An interest rate of 13.9% is required for an investment with continuously compounded interest to double in 5 years.

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Practice

Modeling Real-World Data with Exponential and Logarithmic Functions

Find the amount of time required for an amount to double at the given rate if the interest is compounded continuously.

1. 4.75%

2. 6.25%

3. 5.125%

4. 7.1%

5. ***City Planning*** At a recent town council meeting, proponents of increased spending claimed that spending should be doubled because the population of the city would double over the next three years. Population statistics for the city indicate that population is growing at the rate of 16.5% per year. Is the claim that the population will double in three years correct? Explain.

6. ***Conservation*** A wildlife conservation group released 14 black bears into a protected area. Their goal is to double the population of black bears every 4 years for the next 12 years.
   a. If they are to meet their goal at the end of the first four years, what should be the yearly rate of increase in population?
   b. Suppose the group meets its goal. What will be the minimum number of black bears in the protected area in 12 years?
   c. What type of model would best represent such data?

Practice Problems

Solve the following equations:

Remember that the arguments of all logarithms must be greater than 0. Also exponentials in the form of a^x will be greater than 0. Be sure to check all your answers in the original equation.

1. $3^{x-1} = 81$
2. $8^x = 4$
3. $e^x = 5$
4. $-14 + 3e^x = 11$
5. $-6 + \ln 3x = 0$
6. $\log(3x + 1) = 2$
7. $\ln x - \ln 3 = 4$
8. $2 \ln 3x = 4$
9. $5^{x+2} = 4$
10. $\ln(x + 2)^2 = 6$
11. $4^{-3x} = 0.25$
12. $2e^{2x} - 5e^x - 3 = 0$
13. $\log_7 3 + \log_7 x = \log_7 32$
14. $2 \log_6 4x = 0$
15. $\log_2 x + \log_2(x - 3) = 2$
16. $\log_2(x + 5) - \log_2(x - 2) = 3$
17. $4 \ln(2x + 3) = 11$
18. $\log x - \log 6 = 2 \log 4$
19. $2^x = 64$
20. $5^x = 25$
21. $4^{x-3} = \frac{1}{16}$
22. $3^{x-2} = 81$
23. $\log_3 x = 5$
24. $\log_4 x = 3$
25. $\log_2 2x = \log_2 100$
26. $\ln(x + 4) = \ln 7$
27. $\log_3(2x + 1) = 2$
28. $\log_5(x - 10) = 2$
29. $3^x = 500$
30. $8^x = 1000$
31. $\ln x = 7.25$
32. $\ln x = -0.5$
33. $2e^{0.5x} = 45$
34. $100e^{-0.6x} = 20$
35. $12(1 - 4^x) = 18$
36. $25(1 - e^t) = 12$
37. $\log 2x = 1.5$
38. $\log_2 2x = -0.65$
39. $\frac{1}{3}\log_2 x + 5 = 7$
40. $4 \log_5(x + 1) = 4.8$
41. $\log_2 x + \log_2 3 = 3$
42. $2 \log_4 x - \log_4(x - 1) = 1$