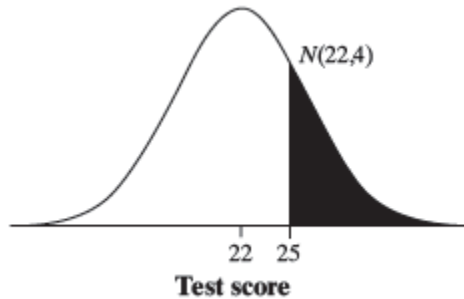


Chapter 2 FRAPPY! Scoring Guidelines

Intent of the question The primary goals of this question are to assess a student's ability to: (1) use a Normal distribution to calculate the proportion of observations in a specified region; (2) use a Normal distribution to determine a specified percentile; (3) determine a linear transformation from summary statistics; (4) justify if a distribution of data is approximately Normal based on summary statistics.

Model Solution

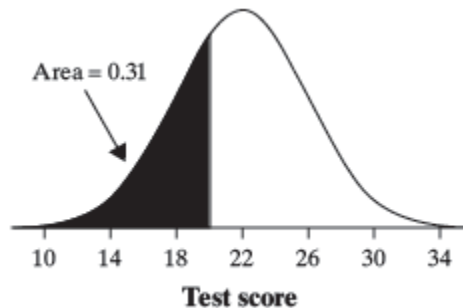
(a) Step 1: State the distribution and the values of interest. The distribution of scores follows a Normal distribution with mean 22 and standard deviation 4. We want to find the proportion of scores that are at least 25.



Step 2: Perform calculations--show your work. The standardized score for the boundary value is $z = \frac{25 - 22}{4} = 0.75$. From Table A, the proportion of z-scores above 0.75 is $1 - 0.7734 = 0.2266$. *Using technology:* The command `normalcdf(lower: 25, upper: 1000, μ : 22, σ : 4)` gives an area of 0.2266.

Step 3: Answer the question. The proportion of students who scored at least 25 points is about 0.23.

(b) Step 1: State the distribution and the values of interest. The distribution of scores follows a Normal distribution with mean 22 and standard deviation 4. The 31st percentile is the boundary value x with 31% of the distribution to its left.



Step 2: Perform calculations--show your work. Look in the body of Table A for a value closest to 0.31. A z-score of -0.50 gives the closest value (0.3085). Solving $-0.50 = \frac{x - 22}{4}$ gives $x = 20$. *Using technology:* The command `invNorm(area: 0.31, μ : 22, σ : 4)` gives a value of 20.02.

Step 3: Answer the question. The 31st percentile is about 20 points.

(c) Because adding a constant does not affect measures of spread:

$$\text{new SD} = b (\text{old SD}) \rightarrow 10 = b (4) \rightarrow b = 2.5$$

Because adding a constant and multiplying by a constant affect measures of center:

$$\text{new mean} = a + b (\text{old mean}) \rightarrow 80 = a + 2.5 (22) \rightarrow a = 25$$

The linear transformation should be: $\text{new score} = 25 + 2.5 (\text{old score})$

(d) If the distribution was Normal, scores should go at least 2 SD above the mean. However, in this distribution, the highest possible score is only $z = (10 - 9.2)/2.1 = 0.38$ standard deviations above the mean. Therefore, it would be inappropriate to use a Normal distribution to do calculations.

Scoring

Parts (a)–(d) are scored essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows

Essentially correct (E) if the response correctly includes the following three components:

1. Indicates use of a normal distribution and clearly identifies the correct parameter values (using a z -score is sufficient);
2. Uses the correct boundary value;
3. Reports the correct normal proportion consistent with components 1 and 2.

Partially correct (P) if the response correctly includes two of the three components listed above.

Incorrect (I) if the response does not satisfy the criteria for an E or a P.

Note: The following are examples of clearly identified parameters for component 1:

- Writes $\mu = 22$ and $\sigma = 4$
- Explicitly labels the mean and standard deviation in a normalcdf calculator statement.
- Sketches a normal curve, labels 22 as the mean, and labels two additional consecutive values separated by 4.

Part (b) is scored as follows

Essentially correct (E) if the response correctly includes the following three components:

1. Indicates use of a normal distribution and clearly identifies the correct parameter values (using a z-score is sufficient);
2. Uses the correct area value;
3. Reports the correct percentile consistent with components 1 and 2.

Partially correct (P) if the response correctly includes two of the three components listed above.

Incorrect (I) if the response does not satisfy the criteria for an E or a P.

Note: The following are examples of clearly identified parameters for component 1:

- Writes $\mu = 22$ and $\sigma = 4$
- Explicitly labels the mean and standard deviation in an invNorm calculator statement.
- Sketches a normal curve, labels 22 as the mean, and labels two additional consecutive values separated by 4.
- Correct identification of parameters in part (a)

Part (c) is scored as follows

Essentially correct (E) if the response calculates both values correctly and shows appropriate work.

Partially correct (P) if the response calculates one of the two values correctly and shows appropriate work.

OR

Calculates both values correctly but does not show appropriate work.

Incorrect (I) otherwise

Note: If the standard deviation is calculated incorrectly and the mean is calculated correctly based on the incorrect standard deviation with appropriate work shown, score the response partially correct (P).

Part (d) is scored as follows

Essentially correct (E) if the response includes the following 3 components

1. States a relevant characteristic of a Normal distribution
2. Uses all three summary statistics (mean, SD, max) to show that the distribution of scores does not share the characteristic of the Normal distribution described in component 1
3. Concludes that the use of a Normal distribution would not be appropriate.

Partially correct (P) if the response includes 2 of the 3 components

Incorrect (I) otherwise.

Notes:

- A response that calculates that approximately 35% of scores should be above 10 in a Normal distribution with $\mu = 9.2$ and $\sigma = 2.1$, states that 0% will be above 10 in the actual distribution, and concludes that a Normal distribution would be inappropriate should be scored essentially correct (E).
- A response that states that it would be inappropriate to use a Normal distribution because the maximum score of 10 is less than 1 standard deviation above the mean should be scored partially correct (P) because it doesn't state a relevant characteristic of a Normal distribution (e.g., "In a Normal distribution, the max should be 2 or 3 standard deviations above the mean.")
- A response that states that it would be inappropriate to use a Normal distribution because 68% of the values should be within 1 standard deviation of the mean (or 95% should be within 2 standard deviations of the mean) should be scored partially correct (P) because the response doesn't show how the actual distribution doesn't share this characteristic.

Each essentially correct (E) part counts as 1 point. Each partially correct (P) part counts as $\frac{1}{2}$ point. If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to decide whether to score up or down, depending on the overall strength of the response and communication, particularly in parts (a) and (d).

- | | |
|---|----------------------|
| 4 | Complete Response |
| 3 | Substantial Response |
| 2 | Developing Response |
| 1 | Minimal Response |