Chapter 10 - Limits

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$$\lim_{x \to a} f(x) = L$$

A number (x-value) → a

The limit of f(x) as x approaches a

A number (y-value) → L

Chapter 10 - Limits

$$Find: \quad \lim_{x \to 4} 3x^2$$

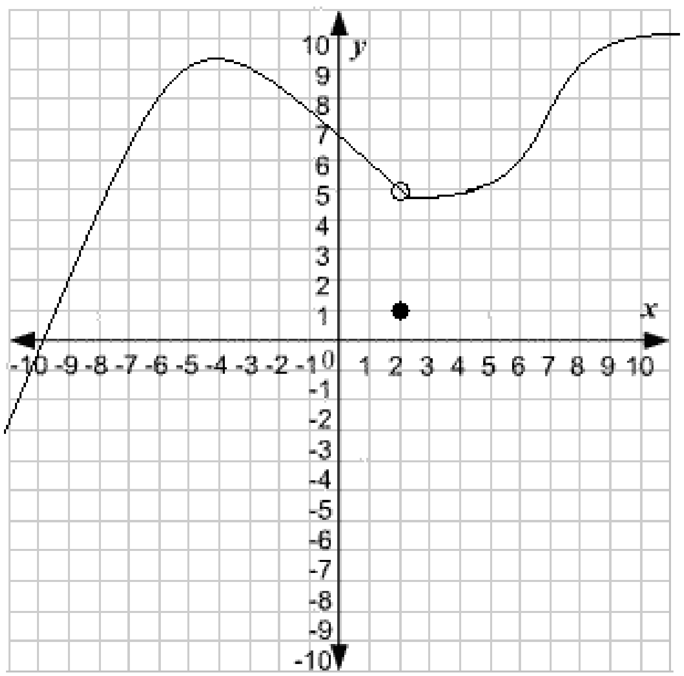
Graphically/Table:
Since $3x^2$ is a continuous function
And there is no error at $x = 4$, we just look at the table
At $x = 4, y = 48$

Algebraically:
Since $3x^2$ is a continuous function
We can just plug and chug

$$3(4)^2 = 48$$

Chapter 10 - Limits
Example

$$\textit{Use the graph to find } \lim_{x \to 2} f(x)$$



Since there is a hole a $x = 2$

(Left) $\lim_{x \to 2^-} f(x)$

You go from left to right and figure out the y-value is 3

(Right) $\lim_{x \to 2^-} f(x)$

You go from right to left and figure out the y-value is 3

Since the left and right match then the answer is 3

Discontinuities

- Hole (removable)
  - there is always a limit at a x-value where there is a hole
- Jump
  - there is sometimes a limit at that x-value
- Asymptotes (non-removable)
  - there is never a limit at an asymptote

Evaluate the following:

$$\lim_{x \to 5} \frac{5-x}{x^2-25}$$

A. $\frac{1}{10}$

B. $\frac{-1}{10}$

C. 0

D. ∞

Algebraically:

First we want to factor, $\frac{5-x}{x^2-25}$

$$\frac{-(x-5)}{(x-5)(x+5)}$$

$$\frac{-1}{x+5}$$

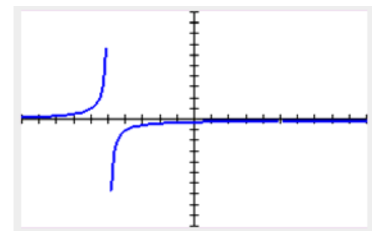
Then plug in, $x = 5$

$$\frac{-1}{5+5} = \frac{-1}{10}$$

The answer is B.

Graphically/Table:

We graph the function $\frac{5-x}{x^2-25}$



Since there is no asymptote at $x = 5$, we go to the table. Since there is an error at $x = 5$ so we look at the number to the left and the right and we also notice that if we follow the pattern then the answer should be B.

X	Y1
2	$\frac{-1}{7}$
3	$\frac{-1}{8}$
4	$\frac{-1}{9}$
5	ERROR
6	$\frac{-1}{11}$
7	$\frac{-1}{12}$

X=5

or

X	Y1
2	-.1429
3	-.125
4	-.1111
5	ERROR
6	-.0909
7	-.0833
8	-.0769
9	-.0714
10	-.0667
11	-.0625
12	-.0588

X=5

Algebraically:
Factor first

$$\frac{\cancel{x+2}}{(\cancel{x+2})(x-2)}$$

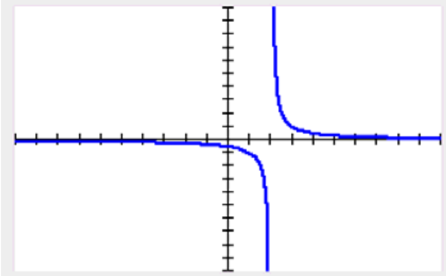
Since $x+2$ is the removable one then $x=-2$ is the hole, so a limit exists. The answer is C.

Luis was evaluating the following function:

$$f(x) = \frac{x+2}{x^2-4}$$

He determined there were discontinuities at $x = \pm 2$. Which of the following correctly identifies the justification for the discontinuity?

Graphically:
Plug it in and see which one is NOT an asymptote.
Which is $x = -2$ so the answer is C.



A. $x = 2$ is removeable because $\lim_{x \to 2} f(x)$ exists.

~~B. $x = 2$ is non-removeable because $\lim_{x \to 2} f(x)$ exists.~~

C. $x = -2$ is removeable because $\lim_{x \to -2} f(x)$ exists.

~~D. $x = -2$ is non-removeable because $\lim_{x \to -2} f(x)$ exists.~~

Not possible because the limit does not exist for non-removable

$$\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$$

Does the limit exist at x = 1 if so what is it?

Algebraically:

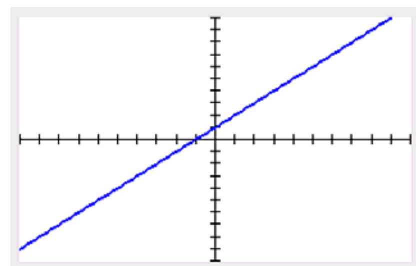
Factor first

$$\frac{(x+1)\cancel{(x-1)}}{\cancel{(x-1)}}$$

Since $x - 1$ is the removable one then $x = 1$ is the hole, so a limit exists. So you plug $x = 1$ in $x + 1$, which is 2.

Graphically/Table:

Plug it in and since there is no asymptote we go to the table. From the pattern we can see that at $x = 1$, the answer should be 2.



X	Y1
-1	0
0	1
1	ERROR
2	3
3	4
4	5
5	6
6	7
7	8
8	9
9	10

X= -1

Extreme Value Theorem

IF A GRAPH IS NOT GIVEN THEN IT SHOULD ALL BE DONE ON CALCULATOR

<u>Maximum/Minimum</u>

- Relative
- Absolute
- Global
- Local

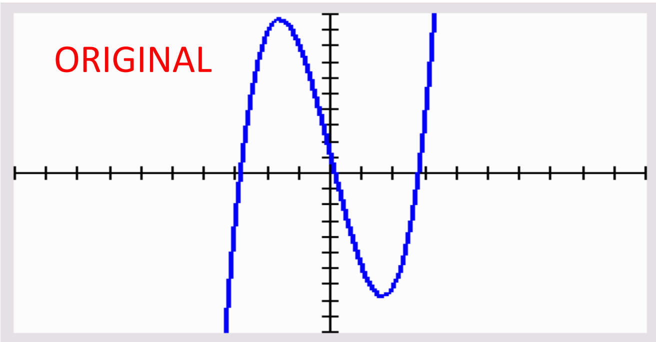
Vocab that describes max and min

Absolute and Global are interchangeable
These are the maximum of maximums and minimum of minimums

Relative and Local are interchangeable
These are not absolute/global but are max/min

The reason they are saying local is because the graph goes on and on

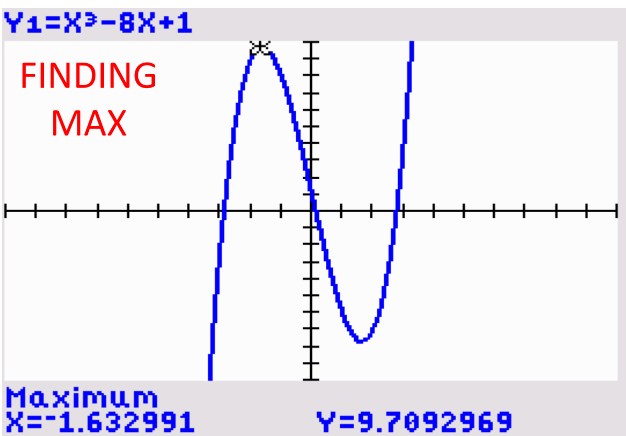
Find the local maximum and minimum values of

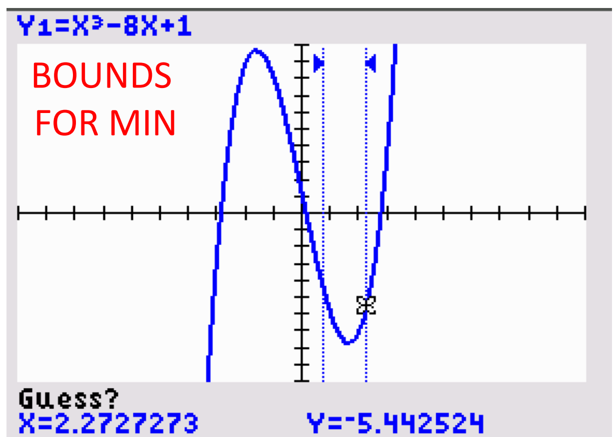


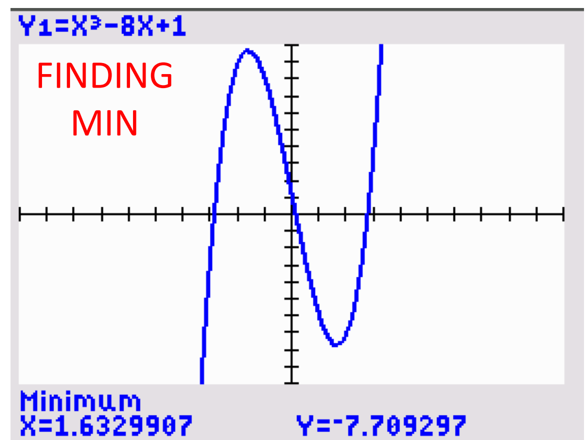
$$f(x) = x^3 - 8x + 1$$

Graphically:
Plug into calculator
Click on 2nd Calc then go to max/min
Make sure bounds are correct

Local Max at 9.7092
Local Min at -7.7092





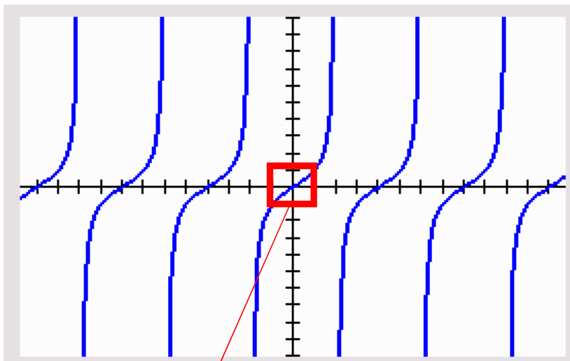


Example: Use the Extreme Value Theorem to decide whether f(x)=tan(x) has a minimum and maximum on the interval $[\frac{-\pi}{4}, \frac{\pi}{4}]$. What about on the interval $[-\pi, \pi]$? Explain your reasoning.

Radians

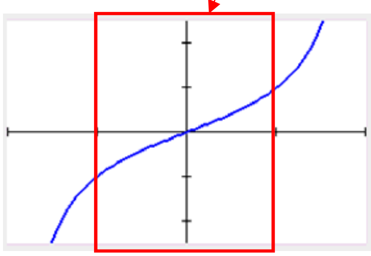
Must fix window spacing

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=π/4
 Ymin=-10
 Ymax=10
 Yscl=1
```



We are only looking from $\frac{-\pi}{4}$ to $\frac{\pi}{4}$. The table will be more accurate. 2nd TBLSET, change to independent to ASK

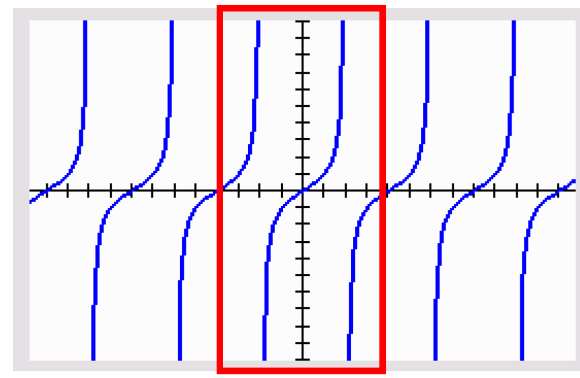
```
TABLE SETUP
 TblStart=-1
 ΔTbl=1
Indpnt: Auto Ask
Depend: Auto Ask
```



$[\frac{-\pi}{4}, \frac{\pi}{4}]$. Max: 1
Min: -1

X	Y_1
.7854	1

X= -π/4

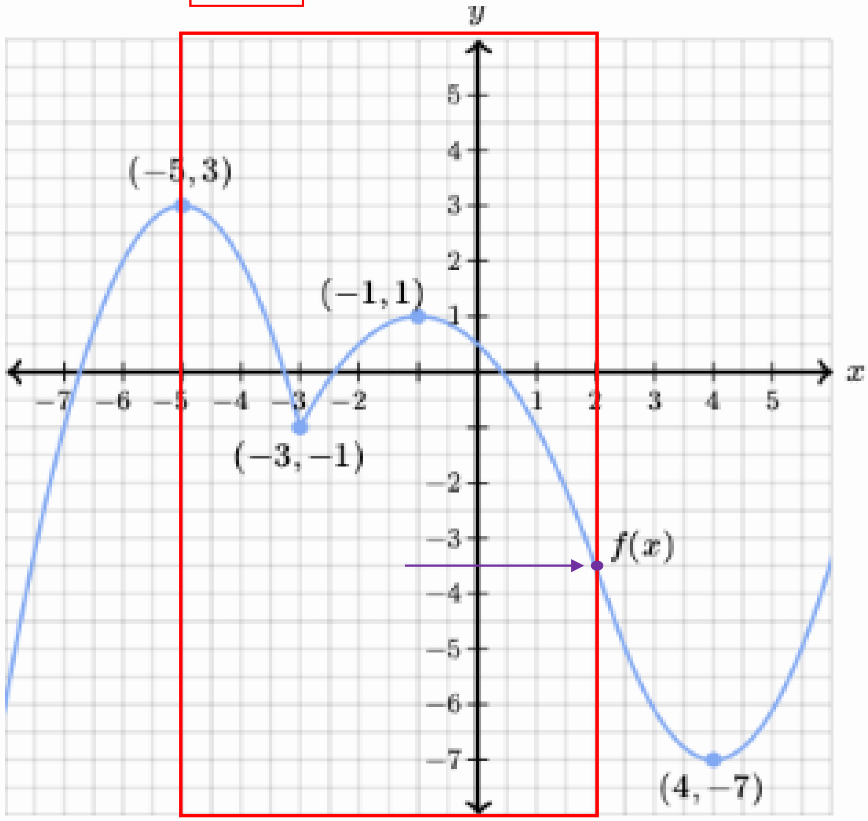


Four dashes is $4(\frac{\pi}{4})$ which is π
Notice that there are asymptotes within the interval so the max/min goes on and on.

$[-\pi, \pi]$. Max: ∞
Min: -∞

The graph of the continuous function $f(x)$ is shown.

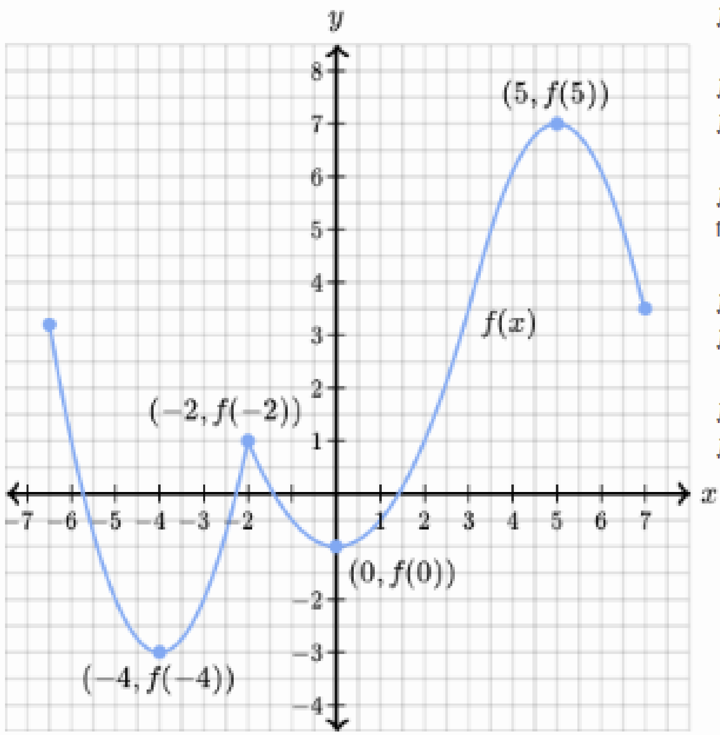
On the interval $x \in [-5, 2]$, what are the extreme values of $f(x)$?



- Local Max: 1
- Local Min: -1
- Absolute Max: 3
- Absolute Min: -3.5

Consider the graph of the function f which is continuous for $x \in [-6.5, 7]$.

With the graph as an aid, answer the questions below.



You never say absolute on an open interval

$f(-4)$ is called a(n) local min because -4 is in the open interval $I = (-6, -2)$ and $f(-4) \leq f(x)$ for all $x \in I$.

$f(-2)$ is called a(n) local max because -2 is in the open interval $I = (-4, 0)$ and $f(-2) \geq f(x)$ for all $x \in I$.

$f(0)$ is called a(n) local min because 0 is in the open interval $I = (-2, 2)$ and $f(0) \leq f(x)$ for all $x \in I$.

$f(-4)$ is called the absolute min because -4 is in the closed interval $I = [-6.5, 7]$ and $f(-4) \leq f(x)$ for all $x \in I$.

You can have absolute and local on a closed interval

$f(5)$ is called the absolute max because 5 is in the closed interval $I = [-6.5, 7]$ and $f(5) \geq f(x)$ for all $x \in I$.