# Introduction to Combinatorics

# The Fundamental Principle of Counting

- The total number of possible outcomes of a series of decisions (i.e. making selections from various categories) is found by multiplying the number of choices for each decision (or category).
- Example1: The Pick – 3 Lottery.
  - How many possible Pick – 3 numbers are there? (You can repeat digits).
  - Each digit in the Pick – 3 ranges from 0 to 9, thus there are 10 digits to choose from.
  - Therefore, there 10 x 10 x 10 = 1000 Pick – 3 numbers.

# Example 2: Arizona License plates

- How many possible Arizona license plates are there given that the first 3 places on the plate are letters (Caps only) and the last 3 places are digits (0 thru 9)?
- Answer: Since there are 6 places on a AZ plate we have 6 categories. 3 categories are letters 3 are digits. Since there are 26 letters and 10 digits the answer is:
- 26 x 26 x 26 x 10 x 10 x 10 = 17,576,000.

# Example 3: Number of rows in a truth table.

- Each simple statement has 2 choices true or false.
- Let's say I have one simple statement, then that statement is true or false, hence a truth table with 2 rows.
- If I have two simple statements then each statement can be true or false so the number of rows would be 2 x 2 = 4
- If I have three simple statements then the number of rows is 2 x 2 x 2 = 8

# Factorials

- Factorials are a short-hand way of writing out a string of multiplications.
- The symbol for factorial is the exclamation point !
- Five factorial is $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$
- If n is a positive integer, then n factorial is defined as

  $$n! = n \times (n - 1) \times (n - 2) \ldots \times 2 \times 1.$$

- As a special case $0! = 1$.

# Examples of factorials

- 0! = 1
- 1! = 1
- 2! = 2 x 1 = 2
- 3! = 3 x 2 x 1 = 6
- 4! = 4 x 3 x 2 x 1 = 24
- 5! = 5 x 4 x 3 x 2 x 1 = 120
- 6! = 6 x 5 x 4 x 3 x 2 x 1 = 720
- 7! = 7 x 6 x 5 x 4 x 3 x 2 x 1 = 5,040
- 8! = 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 40,320
- 9! = 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 362,880
- 10! = 10 x 9 x 8 x 7 x 6 x 5 x 4 x 3 x 2 x 1 = 3,628,800
- Wow! They start out slow but get big very fast.

# Factorial Arithmetic

Compute the following:

1. (7!)(3!) = (5040)(6) = 30,240 Note this is **NOT** 21! = 51,090,942,171,709,440,000. I believe you would say this as 51 quintillion, 90 quadrillion, 924 trillion, 171 billion, 709 million, 440 thousand.
2. $\frac{11!}{5!} = \frac{11\times10\times9\times8\times7\times6\times5\times4\times3\times2\times1}{5\times4\times3\times2\times1}$

   Notice that you get some cancellation.

   You are left with $\frac{11!}{5!} = 11\times10\times9\times8\times7\times6 = 332{,}640$