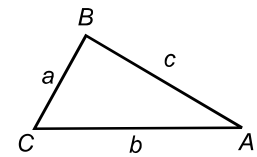
Study Guide

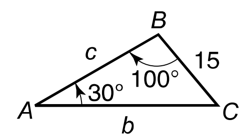
The Law of Sines

Given the measures of two angles and one side of a triangle, we can use the **Law of Sines** to find one unique solution for the triangle.



Law of Sines	$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

Example 1 **Solve $\triangle ABC$ if $A = 30°$, $B = 100°$, and $a = 15$.**



First find the measure of $\angle C$.
$C = 180° - (30° + 100°)$ or $50°$

Use the Law of Sines to find b and c.

$$\frac{a}{\sin A} = \frac{b}{\sin B} \qquad \frac{c}{\sin C} = \frac{a}{\sin A}$$

$$\frac{15}{\sin 30°} = \frac{b}{\sin 100°} \qquad \frac{c}{\sin 50°} = \frac{15}{\sin 30°}$$

$$\frac{15 \sin 100°}{\sin 30°} = b \qquad c = \frac{15 \sin 50°}{\sin 30°}$$

$$29.54423259 \approx b \qquad c \approx 22.98133329$$

Therefore, $C = 50°$, $b \approx 29.5$, and $c \approx 23.0$.

The area of any triangle can be expressed in terms of two sides of a triangle and the measure of the included angle.

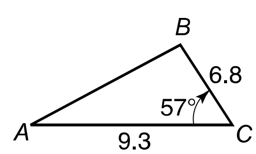
Area (K) of a Triangle	$K = \frac{1}{2}bc \sin A$ $K = \frac{1}{2}ac \sin B$ $K = \frac{1}{2}ab \sin C$

Example 2 **Find the area of $\triangle ABC$ if $a = 6.8$, $b = 9.3$, and $C = 57°$.**

$K = \frac{1}{2}ab \sin C$

$K = \frac{1}{2}(6.8)(9.3) \sin 57°$

$K \approx 26.51876336$



The area of $\triangle ABC$ is about 26.5 square units.

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Practice

The Law of Sines

Solve each triangle. Round to the nearest tenth.

1. $A = 38°, B = 63°, c = 15$

2. $A = 33°, B = 29°, b = 41$

3. $A = 150°, C = 20°, a = 200$

4. $A = 30°, B = 45°, a = 10$

Find the area of each triangle. Round to the nearest tenth.

5. $c = 4, A = 37°, B = 69°$

6. $C = 85°, a = 2, B = 19°$

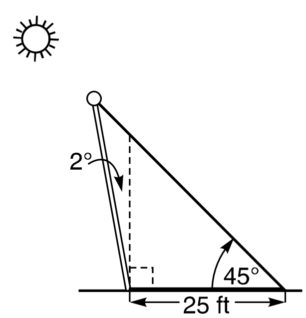
7. $A = 50°, b = 12, c = 14$

8. $b = 14, C = 110°, B = 25°$

9. $b = 15, c = 20, A = 115°$

10. $a = 68, c = 110, B = 42.5°$

11. ***Street Lighting*** A lamppost tilts toward the sun at a 2° angle from the vertical and casts a 25-foot shadow. The angle from the tip of the shadow to the top of the lamppost is 45°. Find the length of the lamppost.



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Study Guide

The Ambiguous Case for the Law of Sines

If we know the measures of two sides and a nonincluded angle of a triangle, three situations are possible: no triangle exists, exactly one triangle exists, or two triangles exist. A triangle with two solutions is called the **ambiguous case**.

Case 1: $A < 90°$ for a, b, and A	
$a < b \sin A$	no solution
$a = b \sin A$	one solution
$a \geq b$	one solution
$b \sin A < a < b$	two solutions
Case 2: $A \geq 90°$	
$a \leq b$	no solution
$a > b$	one solution

Example **Find all solutions for the triangle if $a = 20$, $b = 30$, and $A = 40°$. If no solutions exist, write *none*.**

Since $40° < 90°$, consider Case 1.
$b \sin A = 30 \sin 40°$
$b \sin A \approx 19.28362829$
Since $19.3 < 20 < 30$, there are two solutions for the triangle.
Use the Law of Sines to find B.

$$\frac{20}{\sin 40°} = \frac{30}{\sin B} \qquad \frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\sin B = \frac{30 \sin 40°}{20}$$

$$B = \sin^{-1}\left(\frac{30 \sin 40°}{20}\right)$$

$$B \approx 74.61856831$$

So, $B \approx 74.6°$. Since we know there are two solutions, there must be another possible measurement for B. In the second case, B must be less than 180° and have the same sine value. Since we know that if $\alpha < 90$, $\sin \alpha = \sin (180 - \alpha)$, $180° - 74.6°$ or $105.4°$ is another possible measure for B. Now solve the triangle for each possible measure of B.

Solution I

$C \approx 180° - (40° + 74.6°)$ or $65.4°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{20}{\sin 40°} \approx \frac{c}{\sin 65.4°}$$

$$c \approx \frac{20 \sin 65.4°}{\sin 40°}$$

$$c \approx 28.29040558$$

One solution is $B \approx 74.6°$, $C \approx 65.4°$, and $c \approx 28.3$.

Solution II

$C \approx 180° - (40° + 105.4°)$ or $34.6°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{20}{\sin 40°} \approx \frac{c}{\sin 34.6°}$$

$$c \approx \frac{20 \sin 34.6°}{\sin 40°}$$

$$c \approx 17.66816088$$

Another solution is $B \approx 105.4°$, $C \approx 34.6°$, and $c \approx 17.7$.

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Practice

The Ambiguous Case for the Law of Sines

Determine the number of possible solutions for each triangle.

1. $A = 42°, a = 22, b = 12$

2. $a = 15, b = 25, A = 85°$

3. $A = 58°, a = 4.5, b = 5$

4. $A = 110°, a = 4, c = 4$

Find all solutions for each triangle. If no solutions exist, write none. Round to the nearest tenth.

5. $b = 50, a = 33, A = 132°$

6. $a = 125, A = 25°, b = 150$

7. $a = 32, c = 20, A = 112°$

8. $a = 12, b = 15, A = 55°$

9. $A = 42°, a = 22, b = 12$

10. $b = 15, c = 13, C = 50°$

11. ***Property Maintenance*** The McDougalls plan to fence a triangular parcel of their land. One side of the property is 75 feet in length. It forms a 38° angle with another side of the property, which has not yet been measured. The remaining side of the property is 95 feet in length. Approximate to the nearest tenth the length of fence needed to enclose this parcel of the McDougalls' lot.

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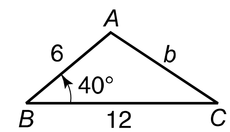
Study Guide

The Law of Cosines

When we know the measures of two sides of a triangle and the included angle, we can use the **Law of Cosines** to find the measure of the third side. Often times we will use both the Law of Cosines and the Law of Sines to solve a triangle.

Law of Cosines	$a^2 = b^2 + c^2 - 2bc \cos A$ $b^2 = a^2 + c^2 - 2ac \cos B$ $c^2 = a^2 + b^2 - 2ab \cos C$

Example 1 **Solve $\triangle ABC$ if $B = 40°$, $a = 12$, and $c = 6$.**



$b^2 = a^2 + c^2 - 2ac \cos B$ *Law of Cosines*

$b^2 = 12^2 + 6^2 - 2(12)(6) \cos 40°$

$b^2 \approx 69.68960019$

$b \approx 8.348029719$

So, $b \approx 8.34$

$\frac{b}{\sin B} = \frac{c}{\sin C}$ *Law of Sines*

$\frac{8.3}{\sin 40°} \approx \frac{6}{\sin C}$

$\sin C \approx \frac{6 \sin 40°}{8.3}$

$C \approx \sin^{-1}\left(\frac{6 \sin 40°}{8.3}\right)$

$C \approx 27.68859159$

So, $C \approx 27.7°$.

$A \approx 180° - (40° + 27.7°) \approx 112.3°$

The solution of this triangle is $b \approx 8.3$, $A \approx 112.3°$, and $C \approx 27.7°$.

Example 2 **Find the area of $\triangle ABC$ if $a = 5$, $b = 8$, and $c = 10$.**

First, find the semiperimeter of $\triangle ABC$.

$s = \frac{1}{2}(a + b + c)$

$s = \frac{1}{2}(5 + 8 + 10)$

$s = 11.5$

Now, apply Hero's Formula

$k = \sqrt{s(s - a)(s - b)(s - c)}$

$k = \sqrt{11.5(11.5 - 5)(11.5 - 8)(11.5 - 10)}$

$k = \sqrt{392.4375}$

$k = \approx 19.81003534$

The area of the triangle is about 19.8 square units.

5-8

Practice

The Law of Cosines

Solve each triangle. Round to the nearest tenth.

1. $a = 20, b = 12, c = 28$

2. $a = 10, c = 8, B = 100°$

3. $c = 49, b = 40, A = 53°$

4. $a = 5, b = 7, c = 10$

Find the area of each triangle. Round to the nearest tenth.

5. $a = 5, b = 12, c = 13$

6. $a = 11, b = 13, c = 16$

7. $a = 14, b = 9, c = 8$

8. $a = 8, b = 7, c = 3$

9. The sides of a triangle measure 13.4 centimeters, 18.7 centimeters, and 26.5 centimeters. Find the measure of the angle with the least measure.

10. ***Orienteering*** During an orienteering hike, two hikers start at point A and head in a direction 30° west of south to point B. They hike 6 miles from point A to point B. From point B, they hike to point C and then from point C back to point A, which is 8 miles directly north of point C. How many miles did they hike from point B to point C?