One-Sided Limits

RECAP Definitions

$$Left-Hand\ Limit: The\ notation$$

$$\lim_{x \to a^-} f(x) = L$$

This means that as *x* gets closer to *a*, but remains <u>less</u> than *a*, the corresponding values of *f(x)* get closer to *L*.

$$Right-Hand\ Limit: The\ notation$$

$$\lim_{x \to a^+} f(x) = L$$

This means that as *x* gets closer to *a*, but remains <u>greater</u> than *a*, the corresponding values of *f(x)* get closer to *L*.

Equal and Unequal One-Sided Limits

- One-sided limits can be used to show that a function has a limit as x approaches a:

$$\lim_{x \to a} f(x) = L \text{ if and only if both}$$

$$\lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = L.$$

The left and right <u>must</u> match for there to be a limit

- One-sided limits can be used to show that a function <u>does not</u> have a limit as x approaches a:

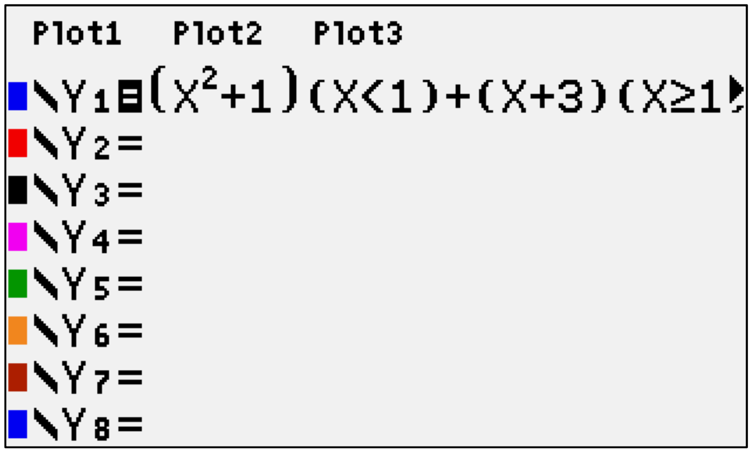
$$\text{If } \lim_{x \to a^-} f(x) = L \quad \text{and} \quad \lim_{x \to a^+} f(x) = M, \text{where } L \neq M,$$

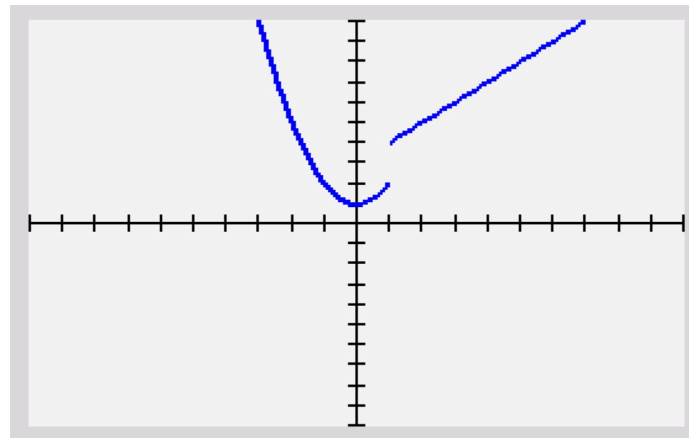
$$\lim_{x \to a} f(x) \text{ does not exist.}$$

If the left and right don’t match then there is no limit

For the function

$$f(x) = \begin{cases} x^2+1 & if\ x<1 \\ x+3 & if\ x\geq 1 \end{cases} \qquad find\ \lim_{x\to 1^+} f(x)$$





2nd →MATH
Is where you will find the < and ≥ symbols

Since we are looking for $x \to 1$ from the <u>right</u> the answer is

$$\lim_{x\to 1^+} f(x) = 4$$

We can confirm this by looking at the table of the separate functions.

For the function

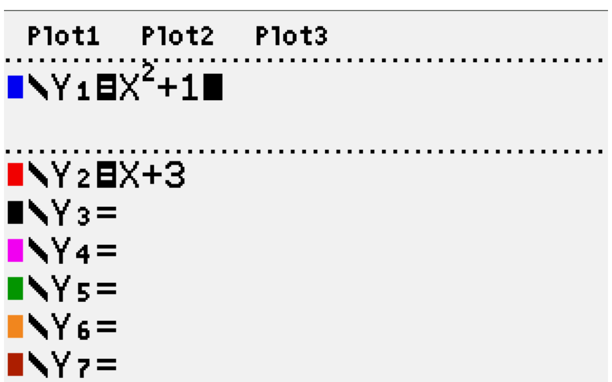
I plugged this into Y1

$$f(x) = \begin{cases} x^2+1 & \text{if } x<1 \\ x+3 & \text{if } x\geq 1 \end{cases} \qquad find \ \lim_{x\to 1^+} f(x)$$

I plugged this into Y2

LOOKING AT THE TABLE INSTEAD OF THE GRAPH

You will have to plug in the equations separately without restrictions



Then look at the table, and evaluate.

Remember smaller numbers means from the left and bigger numbers mean from the right.

X	Y_1	Y_2
0	1	3
1	2	4
2	5	5
3	10	6
4	17	7
5	26	8
6	37	9
7	50	10
8	65	11
9	82	12
10	101	13
X=1	$x<1$	$x\geq 1$

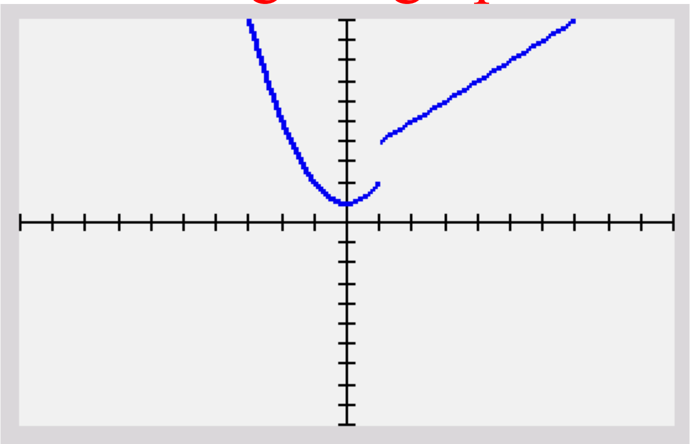
So, according to the table $x = 1$ from the right (red) is equal to 4.

For the function

$$f(x) = \begin{cases} x^2+1 & if\ x<1 \\ x+3 & if\ x\geq 1 \end{cases}$$

$$find\ \lim_{x\to 1^-} f(x)$$

Using the graph



Using the table

X	Y1	Y2		
0	1	3		
1	2	4		
2	5	5		
3	10	6		
4	17	7		
5	26	8		
6	37	9		
7	50	10		
8	65	11		
9	82	12		
10	101	13		

X=1

$$\boxed{\lim_{x\to 1^-} f(x) = 2}$$

For the function

$$f(x) = \begin{cases} x^2+1 & if\ x<1 \\ x+3 & if\ x\geq 1 \end{cases}$$

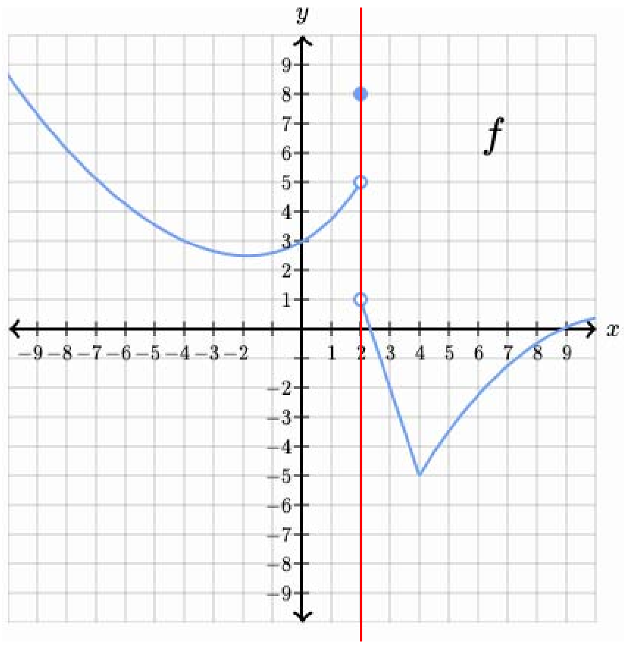
$find\ \lim_{x\to 1} f(x)$

Since we have figured out the two sides we can figure out the actual limit

$\lim_{x\to 1^+} f(x) = 4$ and $\lim_{x\to 1^-} f(x) = 2$

Since the left and right do not match:

For the function below



$A)\ find\ \lim_{x \to 2^-} f(x)$

5

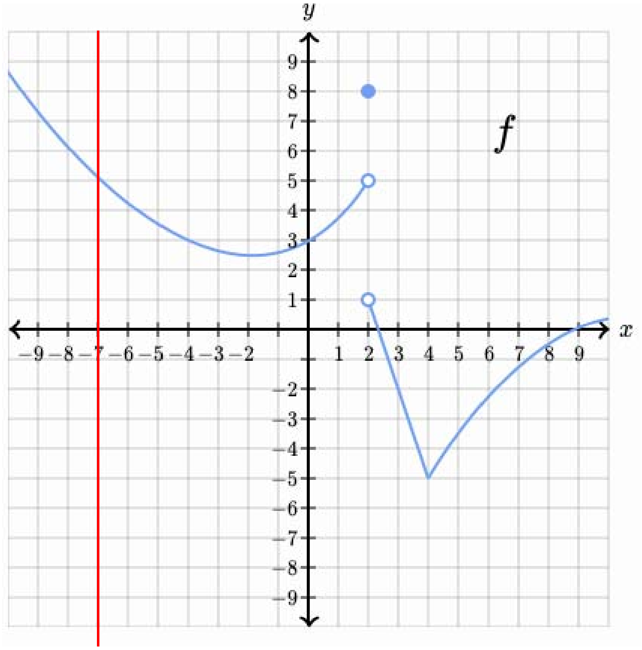
$B)\ find\ \lim_{x \to 2^+} f(x)$

1

$C)\ find\ \lim_{x \to 2} f(x)$

DNE, since left $\neq$ right

For the function below



$A)\ find\ \lim_{x \to -7^-} f(x)$

5

$B)\ find\ \lim_{x \to -7^+} f(x)$

5

$C)\ find\ \lim_{x \to -7} f(x)$

5

Limit by substitution

Find the following limit: $\lim_{x \to -\pi} x$

Just plug it in ☺

$$\lim_{x \to -\pi} x = -\pi$$

Limit by substitution

Find the following limit: $\lim_{x \to 2} \frac{x^3 - 8}{x - 2}$.

Plug it in:

$\lim_{x \to 2} \frac{x^3-8}{x-2} = \frac{2^3-8}{2-2} = \frac{0}{0}$ since it is undefined we must factor!

$$\frac{x^3-8}{x-2} = \frac{(x-2)(x^2+2x+4)}{x-2} = (x^2 + 2x + 4)$$

Plug in $x = 2$:

$$(2)^2+2(2) + 4 = 4 + 4 + 4 = 12$$

Limit by substitution

Find the following limit: $\lim_{x \to 0}(3\cos x)$

Plug it in:

$$\lim_{x \to 0}(3cosx) = 3\cos(0) = 3(1) = 3$$