Limit Properties

Sum, Difference, Product and Quotient

The Limit of a Sum

Remember these represent numbers

$$If \ \lim_{x \to a} f(x) = L \ and \ \lim_{x \to a} g(x) = M, then$$

$$\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x) = L + M.$$

In other words, the limit of the sum of two functions equals the sum of their limits.

The Limit of a Difference

$$If \ \lim_{x \to a} f(x) = L \ and \ \lim_{x \to a} g(x) = M, then$$

$$\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x) = L - M.$$

In other words, the limit of the difference of two functions equals the difference of their limits.

The Limit of a Product

$$If \ \lim_{x\to a} f(x) = L \ and \ \lim_{x\to a} g(x) = M, \ then$$

$$\lim_{x\to a}[f(x) \bullet g(x)] = \lim_{x\to a} f(x) \bullet \lim_{x\to a} g(x) = L \bullet M.$$

In other words, the limit of the product of two functions equals the product of their limits.

The Limit of a Quotient

$$\textit{If } \lim_{x \to a} f(x) = L \textit{ and } \lim_{x \to a} g(x) = M,\ M \neq 0 \textit{ then}$$

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \frac{L}{M},\ M \neq 0.$$

In other words, the limit of the quotient of two functions equals the quotient of their limits, **as long as the limit of the denominator is not zero.**

If the denominator <u>does</u> equal zero, then you <u>must</u> factor the numerator and denominator.

The Limit of a Power

$If \lim_{x \to a} f(x) = L \; and \; n \; is \; a \; positive \; integer \; greater \; than$

$or \; equal \; to \; 2, \; then$

$$\lim_{x \to a}[f(x)]^n = [\lim_{x \to a} f(x)]^n = L^n$$

In other words, the limit of a function to a power is found by taking the limit of the function and then raising this limit to the power.

The Limit of a Root

If $\lim_{x \to a} f(x) = L$ *and n is a positive integer greater than or equal to* 2, *then*

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)} = \sqrt[n]{L}$$

You do the root last!

provided that all roots represent real numbers.
In other words, the limit of the *n*th root of a function is found by taking the limit of the function and then taking the *n*th root of this limit.

Example of Sum

Find: $\lim_{x \to -2} \left(3x^2 + 5x - 9\right)$

$$\lim_{x \to 2} 3x^2 \quad + \quad \lim_{x \to 2} 5x \; - \quad \lim_{x \to 2} 9$$

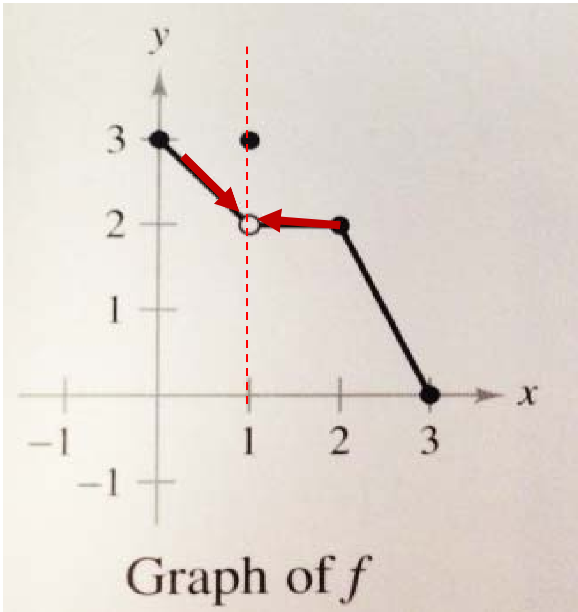
$$3(-2)^2 \quad + \quad 5(-2) - \quad 9$$

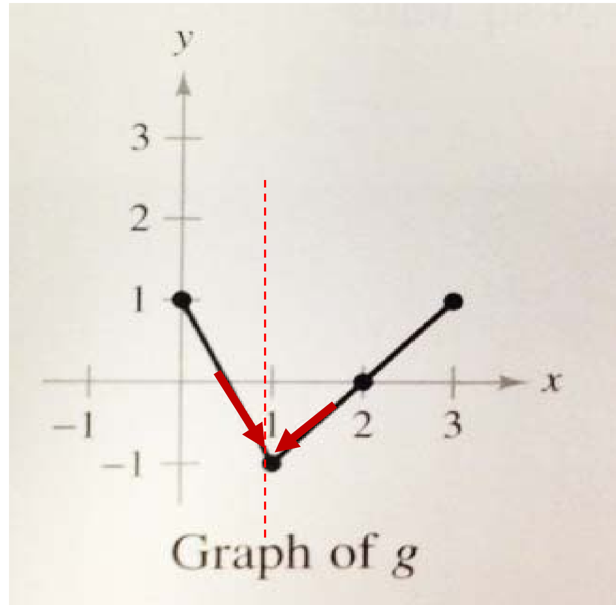
$$3(4) \quad + \quad (-10) - \quad 9$$

$$12 - 10 - 9$$

$$\boxed{-7}$$

Example of Difference



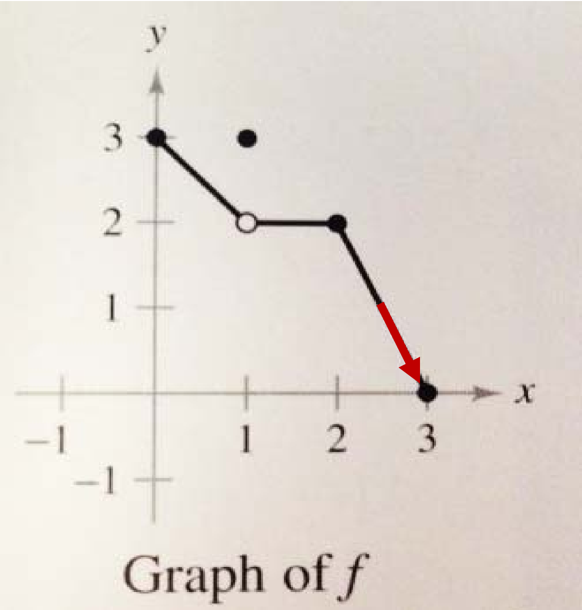


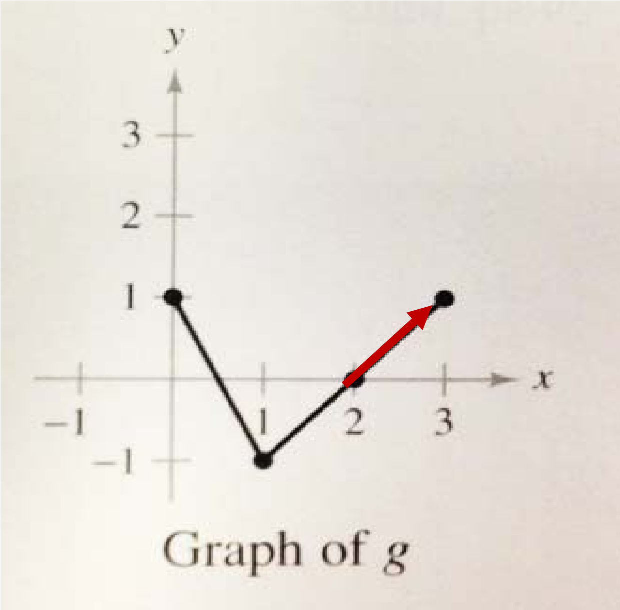
Find: $\lim_{x\to 1}\left(f(x)-g(x)\right)=\lim_{x\to 1} f(x)-\lim_{x\to 1} g(x)$

$2 \quad - \quad (-1)$

$\boxed{3}$

Example of Difference





$$Find: \lim_{x \to 3}\left(g(x) - f(x)\right)$$ = DNE

It is DNE because the limits from the right do not exist

Example of Product

Find: $\lim_{x \to 1}[f(x) \bullet g(x)]$

Let $\lim_{x \to 1} f(x) = 6$ and $\lim_{x \to 1} g(x) = -2$

$$\lim_{x \to 1}[f(x) \bullet g(x)] = \lim_{x \to 1} f(x) \bullet \lim_{x \to 1} g(x)$$

6 • -2

-12

Example of Quotient

Find: $\lim_{z \to 1} \frac{6 - 3z + 10z^2}{-2z^4 + 7z^3 + 1}$

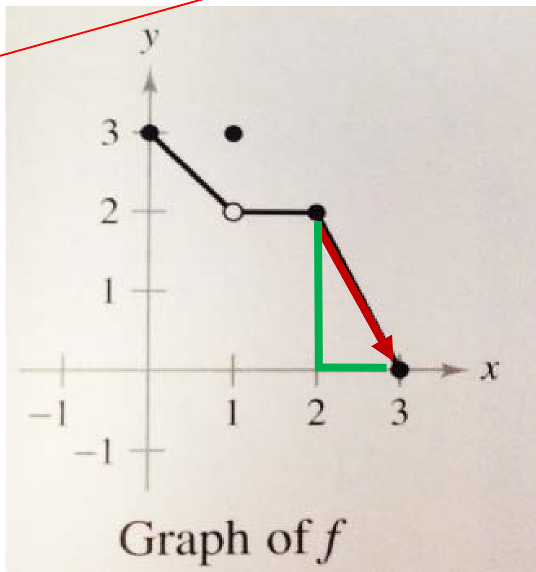
You can do the limit by substitution:

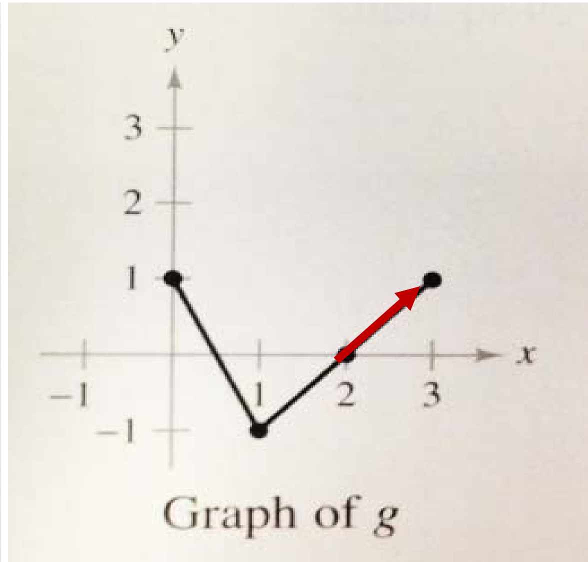
$$\frac{6 - 3(1) + 10(1)^2}{-2(1)^4 + 7(1)^3 + 1} = \frac{6 - 3 + 10}{-2 + 7 + 1} = \boxed{\frac{13}{6}}$$

Example of Quotient

Find: $\lim_{x \to 3^-} \frac{f(x)}{g(x)-1}$ (Assume that f and g are linear on the interval $[2,3]$.)

We are only talking about the left side





First let's just plug it in

$$\frac{f(x)}{g(x)-1} = \frac{0}{1-1} = \frac{0}{0}$$

Since it is undefined, factor!

Since it states that *f* and *g* are linear, we need to figure out their equation.

$y = mx + b$

No, it doesn't matter which point you use

$f(x) = (-2)x + b$	$g(x) = (1)x + b$
$2 = (-2)(2) + b$	$0 = (1)(2) + b$
$2 = -4 + b$	$0 = 2 + b$
$6 = b$	$-2 = b$
$f(x) = (-2)x + 6$	$g(x) = x - 2$

$$\frac{f(x)}{g(x)-1} = \frac{-2x+6}{(x-2)-1} = \frac{-2(x-3)}{x-3}$$

$$= -2$$

Example of Power

$$Find\text{: } \lim_{x\to 3}(3x+1)^2$$

Just plug it in:

$$(3(3)+1)^2 = 10^2 = 100$$

Example of Root

$$Find: \lim_{x\to 3}\sqrt[3]{2x^2+x-10}$$

Plug it in:

$$\sqrt[3]{2(3)^2+(3)-10} = \sqrt[3]{2(9)+(3)-10} = \sqrt[3]{18+(3)-10}$$

$$\boxed{=\sqrt[3]{11}}$$