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Reteaching
8.2 *Rational Functions and Their Graphs*

◆ Skill A Finding the domain of a rational function

Recall Division by zero is not allowed; it is undefined.

◆ Example

Find the domain of $g(x) = \frac{x^2 + 8x}{x^2 + 3x - 10}$.

◆ Solution

You must exclude from the domain any values of x which cause the denominator to have a value of 0.

Set $x^2 + 3x - 10$ equal to 0.

$(x + 5)(x - 2) = 0$

$x = -5$ or $x = 2$

The domain is all real numbers except -5 and 2.

Find the domain of each rational function.

1. $\frac{x + 3}{x^2 - 16}$ ________________

2. $\frac{5x}{x^2 + 7x}$ ________________

3. $\frac{x^2 + 5}{x^2 - 4x - 21}$ ________________

◆ Skill B Identifying vertical asymptotes and holes in the graph of a rational function

Recall If $(x - b)$ is a factor in both the numerator and denominator, there will be a hole in the graph at $x = b$. If $(x - b)$ is a factor of the denominator but not a factor of the numerator, there will be a vertical asymptote of $x = b$.

◆ Example

For the rational function $f(x) = \frac{2x^2 + 3x - 2}{x^2 - x - 6}$

a. identify the x-coordinates of any holes in the graph.

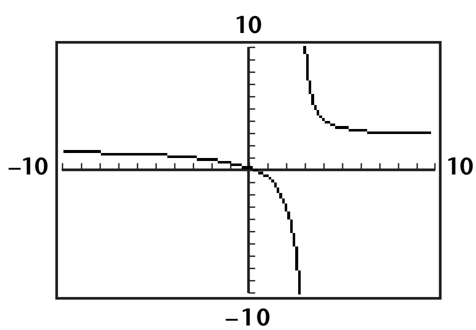
b. write the equations of any vertical asymptotes.

◆ Solution

a. $\frac{2x^2 + 3x - 2}{x^2 - x - 6} = \frac{(2x - 1)(x + 2)}{(x + 2)(x - 3)}$

Since $x + 2$ is a factor of both the numerator and denominator, there will be a hole at $x = -2$.

b. Since $x - 3$ is a factor of the denominator but not the numerator, and has a value of 0 when $x = 3$, there will be a vertical asymptote at $x = 3$.



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Identify all holes and asymptotes in the graph of each rational function.

4. $f(x) = \dfrac{(x - 3)(x + 2)}{(x + 3)(x + 2)}$ ______________________

5. $f(x) = \dfrac{x + 5}{(x - 1)(x + 4)}$ ______________________

6. $f(x) = \dfrac{x^2 - 4x + 3}{x^2 - x - 6}$ ______________________

7. $f(x) = \dfrac{x^2 + 6x - 7}{x - 1}$ ______________________

◆ **Skill C** Writing an equation for the horizontal asymptote of a graph

Recall The degree of a polynomial is the greatest degree of its terms.

◆ **Example**

Write the equation of the horizontal asymptote for each of the following functions.

a. $f(x) = \dfrac{2x^2 + 3x - 2}{x^2 - x - 6}$ **b.** $g(x) = \dfrac{3x}{x^2 - 5}$ **c.** $h(x) = \dfrac{-x^2}{x - 2}$

◆ **Solution**

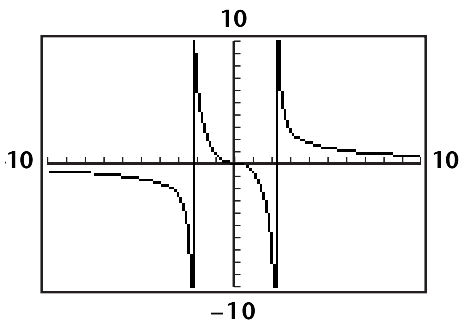
a. Since the degree of the numerator and denominator are the same, divide the coefficient of the term with the greatest degree in the numerator by the coefficient of the like term in the denominator.

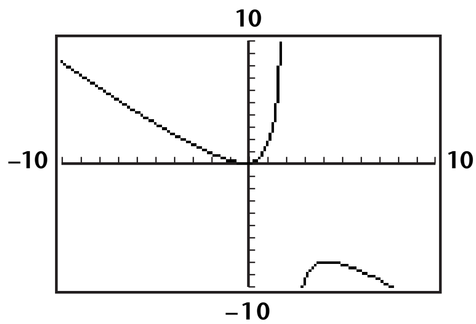
$\dfrac{2}{1} = 2$ (coefficients of the x^2 terms)

Thus, $y = 2$ is a horizontal asymptote. The graph is shown on the preceding page.

b. Since the degree of the numerator is less than the degree of the denominator, the horizontal asymptote is $y = 0$. (The graph is shown at left below.)

c. Since the degree of the numerator is greater than the degree of the denominator, there is no horizontal asymptote. (The graph is shown at right below.)





Identify any horizontal asymptotes for the following functions. Use a graphics calculator to check your answer.

8. $f(x) = \dfrac{5x^2 + 8}{2x^2 - 3x}$ ______________

9. $f(x) = \dfrac{x^2 + 5x - 6}{x + 2}$ ______________

10. $f(x) = \dfrac{x + 5}{(x - 1)(x + 4)}$ ______________